CLIFFORD ALGEBRA REPRESENTATION OF GRASPING AND MANIPULATIVE HAND ACTIONS FOR KINEMATIC SYNTHESIS

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ABSTRACT. The dimensional synthesis of wristed, multi-fingered hands can be used for simultaneous tasks of all fingertips. When defining unconstrained positions for the fingers, the synthesis is solved by ensuring the desired displacement for each branch of the hand. The displacements and related geometric objects are expressed in the even Clifford subalgebra $\mathscr{C}^+_{0,3,1}$. If velocities are also defined, the corresponding Lie algebra elements, defined on the same algebra, are also equated to the linear combination of joint twist; accelerations can be defined in a similar way.

This approach successfully captures the independent motion of each fingertip. However, when the design task includes holding and manipulating an object, the constraints between the fingertips need to be considered too.

In this work the conditions to create a hand motion compatible with grasping and moving a given object, expressed in the Clifford algebra, are shown. The design methodology is outlined and some of the simple grasping and manipulation cases are presented.

1. INTRODUCTION

The design of end-effector robotic tools has focused on three different strategies [10], which yield very different designs: anthropomorphism, designing for grasping tasks, and designing for dexterous manipulation. Hands for in-hand manipulation tend to be more complex, especially if a wide range of manipulation actions are targeted, while underactuation is targeted for grasping and limited manipulation [11], [15], [5]. In this research, we focus on creating multi-fingered hand designs specifically tailored to desired groups of manipulation tasks.

We define a multi-fingered robotic hand as a series of common joints branching at least once in several other serial chains (the fingers or branches), ending in a finite set of end-effector links (the fingertips). Recently a methodology has been developed for the design of new multi-fingered hands for kinematic tasks [20], both for finite and infinitesimal motion [21]. This methodology offers a systematic process to design innovative end-effectors for a simultaneous task of all the fingertips [8]. However the design for manipulation of a grasped objects requires a more careful strategy.

When the hand grasps and object, the constraints on the relative motion among fingers need to be taken into account; the kinematic topology is switched from a tree topology to a hybrid topology. In this application, we focus on a simple type of in-hand manipulation, for a given fingertip contact point [6] and some specified relative motion, compatible with the contact. The mobility of the grasped object can be calculated for the general case using mobility formulas, or using grasp or Jacobian matrix techniques [3] if the hand kinematics and object geometry are at least partially known.

Given a hand topology and the mobility for a generally-grasped object, a series of positions and subspaces of potential velocities are defined so that some properties of the grasp are being checked while allowing the manipulation of the object. These positions and velocities are expressed using the even Clifford subalgebra $\mathscr{C}^+_{0,3,1}$. Several authors have used a Clifford or geometric algebra to define displacements and velocities of rigid bodies an robotic systems, among others [7], [19], [1] or [2]. In particular, the analysis of contacts using Grassman-Cayley algebra was developed in [22], the definition of contacts for synthesis problem of planar linkages was developed in [12], and the analysis and planning of grasping using geometric algebra has been studied in [23].

It is expected that the use of the Clifford algebra will allow a more compact and more homogeneous expression of the grasping and manipulation actions including finite positions and its derivatives, for their use in the design of innovative robotic hands.

2. Displacements, velocities and forces

Let $\mathscr{C}^+_{0,3,1}$ be the even Clifford subalgebra of the projective space $\mathbb{P}^4$ with the degenerated scalar product. Starting with the basis vectors $\{e_1, e_2, e_3, e_4\} \in \mathbb{P}^4$, the well-known notation for the even blades,

$$\begin{aligned} e_{23} &= i, e_{31} = j, e_{12} = k, \\ e_{41} &= i\varepsilon, e_{42} = j\varepsilon, e_{43} = k\varepsilon, \\ e_{1234} &= \varepsilon \end{aligned} \tag{1}$$

is used along this work.

Consider a general element of this algebra as $A = a_0 + a_1 i + a_2 j + a_3 k + \varepsilon(a_4 i + a_5 j + a_6 k + a_7)$, and the geometric product of two 1-vectors as the sum of the inner product and the exterior product, $ab = a \cdot b + a \wedge b$.

The *conjugation* is defined for blades as $(e_1 e_2 \ldots e_k)^* = (-1)^k e_k \ldots e_2 e_1$; for scalars, $1^* = 1$ and for basis vectors, $e_i^* = -e_i$.

The norm of an element is $||A||^2 = a + \varepsilon a^0 = AA^*$ and it has nonzero scalar and dual part. For unit elements, $||A||^2 = 1$. The inverse of an element is defined as $A^{-1} = A^*/||A||^2$, so that for unit elements, such as displacements, the inverse is $A^{-1} = A^*$.

A point is defined as $p = 1 + \varepsilon(p_x i + p_y j + p_z k)$, and a line is defined as $L = l_x i + l_y j + l_z k + \varepsilon(l^0_x i + l^0_y j + l^0_z k)$, being such that $LL^* = 1$.

A finite displacement is a unit element of the subalgebra, and can be expressed as a function of the invariants of the displacement, the screw axis S and the rotation ϕ and slide t about and along the axis. In particular,

- A translation of magnitude d along a direction $\mathbf{s}$ is $D = 1 + \frac{d}{2}\varepsilon(s_x i + s_y j + s_z k)$
- A rotation of magnitude ϕ and rotation axis $\mathbf{s}$ is $R = \cos\frac{\phi}{2} + \sin\frac{\phi}{2}(s_x i + s_y j + s_z k)$
- A general displacement of screw axis $\mathsf{S} = \mathbf{s} + \varepsilon\mathbf{s}^0$, rotation ϕ and slide t is $Q = DR = \cos\frac{\phi}{2} + \sin\frac{\phi}{2}(s_x i + s_y j + s_z k) + \varepsilon((\sin\frac{\phi}{2}s^0_x + \frac{t}{2}\cos\frac{\phi}{2}s_x)i + (\sin\frac{\phi}{2}s^0_y + \frac{t}{2}\cos\frac{\phi}{2}s_y)j + (\sin\frac{\phi}{2}s^0_z + \frac{t}{2}\cos\frac{\phi}{2}s_z)k) - \frac{t}{2}\sin\frac{\phi}{2})$

2.1. **Twists as 2-vectors.**

In order to define the velocities, we consider the differentiation of the action of a finite displacement on a geometric element x expressed in the moving frame,

$$\dot{X} = \dot{Q}xQ^* + Qx\dot{Q}^*, \tag{2}$$

which leads to

$$\dot{X} = (\dot{Q}Q^*)X + X(\dot{Q}Q^*)^*. \tag{3}$$

Define

$$V = 2\dot{Q}Q^*. \tag{4}$$

Recalling that Q needs to be a unit element in order to be a displacement, and taking derivatives in the unit condition we obtain that $\dot{Q}Q^* = -(\dot{Q}Q^*)^*$, which makes V a pure element, that is, an element with zero scalar and pseudoscalar components. This element is also denoted a *2-vector*, as it is a linear combination of 2-blades only.

Using the definition in Eq.(4), the derivative becomes

$$\dot{X} = \frac{1}{2}(VX + XV^*). \tag{5}$$

The definition in (4) can be arranged as

$$\dot{Q} = \frac{1}{2}VQ, \tag{6}$$

which coincides with that in [7]. The element $V = w + \varepsilon v$ is the *twist*, which describes the velocity of the rigid body, as the angular velocity of the body and the linear velocity of a point of the body. For the calculations above, the point is the origin of the fixed frame as considered part of the moving body.

If $s = s_x i + s_y j + s_z k$ and $s^0 = s^0_x i + s^0_y j + s^0_z k$, the computation of V using the expression of displacement Q in (4) yields

$$V = \dot{\phi}J = \dot{\phi}(s + \varepsilon(s^0 + hs)). \tag{7}$$

Here $\mathsf{S} = \mathbf{s} + \varepsilon\mathbf{s}^0$ is the line defining the screw axis of the displacement, corresponding to the minimal motion for the finite displacement. The magnitude $h = \frac{\dot{t}}{\dot{\phi}}$ is called the *pitch*. This 2-vector that contains both the angular velocity of the rigid body and the linear velocity of the origin point, can be immediately identified with the six-dimensional twist $\mathsf{V} = (\omega, \mathbf{v})$ defined in screw theory, while J would correspond to the unit twist, or screw. [19].

Due to the different action used for lines and points (and planes), the conjugate 2-vector in Eq.(5) becomes, for directions and lines,

$$V^* = \dot{\phi}J^* = \dot{\phi}(-s - \varepsilon(s^0 + hs)), \tag{8}$$

and for points and planes,

$$V^* = \dot{\phi}(-s + \varepsilon(s^0 + hs)), \tag{9}$$

The Lie algebra $se(3)$ can be built on this geometric algebra if we consider the 2-vectors and define the commutator product of two elements V_1, V_2 of the algebra as

$$[V_1, V_2] = \frac{1}{2}(V_1V_2 - V_2V_1), \tag{10}$$

which is closed for the pure elements. Using the commutator, we can define

$$V_1V_2 = [V_1, V_2] + V_1 \wedge V_2 + V_1 \cdot V_2. \tag{11}$$

Notice that $[V_1, V_2]$ yields the dual cross product used in the dual vector calculus [18], while the exterior and inner product yield the minus dual dot product.

Integrating Eq.(6) and considering V as constant, we obtain the finite displacement as the exponential of the twist, defined as a power series, starting at the identity.

$$Q = e^{\frac{1}{2}Vt} = e^{J\frac{\phi}{2}} \tag{12}$$

if we consider $\dot{\phi} = \phi/t$.

This derivation can be found in detail for instance in [13] and it is used to create the forward kinematics of serial robots as the Clifford product of exponentials.

2.2. **Linear property of 2-vectors.** The Clifford algebra is not a graded multi vector algebra, however it is a graded vector space [4]: it can be decomposed as sum of linear subspaces of homogeneous grade. The twists, or 2-vectors, form a vector subspace within the Clifford algebra over the scalars (0-vectors) and also over the *dual scalars*, which are the multi vectors constructed with elements of degree zero and degree four, $K = k_0 + \varepsilon k_7$ [14]. The addition and product by scalar and pseudoscalar elements yields a 2-vector in all cases,

$$K_1V_1 + K_2V_2 = k_{10}w_1 + k_{20}w_2 + \varepsilon(k_{10}v_1 + k_{20}v_2 + k_{17}w_1 + k_{27}w_2). \tag{13}$$

In summary, considering either scalars or dual scalars, the 2-elements, which we identify with screws for both twists and wrenches, can form vectors subspaces in the Clifford algebra. We can see the finite screw systems as vector subspaces formed by elements of degree 2 of the Clifford algebra.

2.3. **Wrenches as reciprocal screws.** The 2-vectors are used to express the twist defined as before, $W = w + \varepsilon v$ and also the wrench $F = m + \varepsilon f$, where $m = m_x i + m_y j + m_z k$ is the resultant moment and $f = f_x i + f_y j + f_z k$ is the resultant force at a given point of the rigid body. The reciprocal product is defined in screw theory as $W * F = w \cdot m + v \cdot f$; when this scalar quantity is zero, it is said that the twist and the wrench are reciprocal.

Given a wrench or wrench subspace, representing the contact forces on a body, there exists a reciprocal subspace of twists for the potential velocities allowed for the body. This reciprocity, which exists at the level of first derivatives and for convex and polygonal objects, is used in this work to define the grasping and manipulation actions as a function of the twists.

The inner and outer products in Eq.(11) yield the scalar and pseudoscalar,

$$\begin{aligned} W \cdot F &= -\mathbf{w} \cdot \mathbf{m}, \\ W \wedge F &= -(\mathbf{w} \cdot \mathbf{f} + \mathbf{v} \cdot \mathbf{m})\varepsilon \end{aligned} \tag{14}$$

3. Kinematics of grasping for tree topologies

3.1. **End-effector twist and Jacobian.** Let S_1 to S_n be the ordered n axes of a serial chain, and $J_1, \dots, J_n$ the corresponding unit screws, in which the pitch $h = t/\theta$ is used to identify either a prismatic or a revolute joint. The product of exponentials

$$Q = e^{\frac{\theta_1}{2} J_1} \dots e^{\frac{\theta_n}{2} J_n} \tag{15}$$

yields the relative motion of the rigid body attached to the last joint (the end effector) with respect to a reference configuration. These are the relative forward kinematics equations. The Jacobian matrix of the serial chain can be derived by finding the twist of the end effector as in (4), noticing that in the product of exponentials, only the joint variables θ_i are a function of time,

$$V = 2\dot{Q}\tilde{Q} = 2\Big(\sum_{i=1}^{n} \frac{\partial Q}{\partial \theta_i}\dot{\theta}_i\Big)\tilde{Q} = 2\sum_{i=1}^{n} \Big(\frac{\partial Q}{\partial \theta_i}\tilde{Q}\Big)\dot{\theta}_i. \tag{16}$$

If we expand the above calculation, we obtain that for each axis,

$$(\frac{\partial Q}{\partial \theta_i}\tilde{Q})\,\dot{\theta}_i = \frac{1}{2}e^{J_1\frac{\theta_1}{2}}\ldots e^{J_{i-1}\frac{\theta_{i-1}}{2}}J_i e^{-J_{i-1}\frac{\theta_{i-1}}{2}}\ldots e^{-J_1\frac{\theta_1}{2}}\dot{\theta}_i \tag{17}$$

Notice that this is the action of the displacement of the preceding joints, up to the $i-1^{th}$ joint, on the i^{th} joint, for the current configuration of the serial chain. Denote as $J_i' = Q_{i-1}J_i\tilde{Q}_{i-1}$ the current position of the screw, with Q_{i-1} being the displacement due to the previous joints to joint i; the end effector twist becomes

$$V = \sum_{i=1}^{n} J_i'\dot{\theta}_i. \tag{18}$$

Notice that the expression of the screw J_i' is parameterized by the previous $i-1$ joint variables.

The twist vector V is a linear combination of unit twists times scalar joint rates, and contains the feasible velocities and angular velocities of the end-effector with respect to the fixed frame. When written in matrix form with the screws as columns, they form the *fixed-frame Jacobian* or *spatial Jacobian* matrix of the serial robot.

When the Jacobian matrix is to be created for a particular geometric entity X, consider $\dot{X} = \frac{1}{2}(VX + XV^*) = V \times X$, which yields

$$\dot{X} = (\sum_{i=1}^{n} J_i'\dot{\theta}_i) \times X = \sum_{i=1}^{n}(J_i' \times X)\dot{\theta}_i \tag{19}$$

with the columns of the Jacobian matrix modified for the particular geometric element to consider. The derivation of the Jacobian can be found for instance in [1]. For wristed robotic hands with a tree topology, the same derivation can be made, in which some of the joints are common to some of the branches.

3.2. **Grasp analysis.** Grasping consists on locating several end effectors in contact with the surface of an object, so that the forces applied at the contact points ensure some desired resultant force property on the object, force closure being one of them. A grasp is *force closed* if it can balance any external wrench applied at the object. In order to analyze the grasping, it is then important to look at the static forces exerted or their reciprocal potential velocities.

Let V_i be the 2-vector for the twist of the end effector i, and F_i the reciprocal 2-vector corresponding to the wrench of the end effector i, and let us consider b end effectors able to exert contact forces on the object. Then we need to impose

$$\sum_{i=1}^{b} c_i F_i + F = 0, \tag{20}$$

where F is the external wrench on the object, and $c_i \geq 0$ are the scalars defining a positive grasp, all transformed to either the fixed frame or to an object frame. Figure 1 shows the typical local finger frame and the object frame in which the force balance is stated.

The force closure can be expressed using reciprocal twists. A *form-closed grasp* is that in which the space of feasible velocities is zero. Form and force-closed grasps coincide for polyhedral, convex objects; for other cases, the curvature of the object needs to be taken into account for the form-closed grasp.

For a pointy finger with no friction, we define a local frame such that $F_i = c_i k\varepsilon$ with $c_i \geq 0$. In this frame, the z-axis is pointing towards the object, as shown in Figure 1. It has been proved that for a general object, seven fingers are needed with positive forces

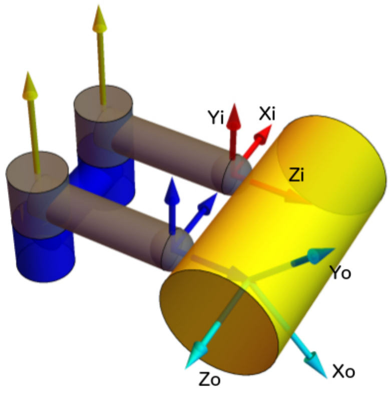

FIGURE 1. Two fingers in contact with a convex object and their respective finger frames.

in order to ensure force-closed grasp. Obviously, if the scalar c_i is allowed to take any value, then six fingers are needed to balance any external wrench. When friction is considered, the number of fingers can be further reduced.

The wrench of each finger can be transformed to an object frame (located for instance at the center of mass) as $F_{oi} = Q_i F_i Q_i^*$, with Q_i the displacement from the finger frame to the object frame.

3.3. **Kinematic model of a grasping hand.** The definition of the number and position of the contact points and the contact forces created on the object allows us to analyze several aspects of the grasp. Assuming that some grasp synthesis method (see for instance [16]) is used to compute those points, then the next step is to analyze wether the end-effectors of the robotic hand (usually the fingertips) can reach the desired positions and whether the needed forces can be applied.

If a robotic hand is represented as a tree graph as shown in Figure 2, then the robotic hand grasping an object becomes a hybrid graph, in which the fingertip contacts can be modeled as different types of joints. In Figure 2, the robotic hand in the left has five fingertips, two palms and a wrist. The numbers on the edges denote the number of joints in the serial chain. The graph in the right shows the same hand grasping an object (square box) with the five fingertips and with contact also in one of the palms. The contact joint is denoted as F.

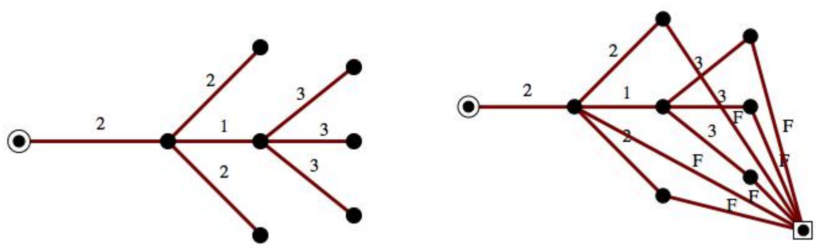

FIGURE 2. A five-fingered robotic hand, left; grasping an object, right.

Several fingertip contacts are considered, depending on the modeling of the friction. The simplest fingertip is what is called the *pointy finger*, in which a point contact with no friction is considered. The description of the different fingertip contacts can be found for instance in [17].

Mason [9] defines, following Salisbury, contact types and the mobility and connectivity (relative mobility of two links) of a robotic hand with several fingers using the well-known Kutzbach-Gruebler formula, M for the mobility when the finger joints are allowed to move, and M' for the mobility when the finger joints are locked, which will give the subspace of twists of the object when the hand is trying to immobilize the object in a grasp. In addition we define M_w as the mobility of the hand minus the degrees of freedom of the common joints up to the first split. These values are used a priori to select appropriate hand topologies for a given task.

4. GRASPING AND MANIPULATION ACTIONS FOR SYNTHESIS

For this work we consider convex objects and spherical fingertips, and use twists in order to define the grasp as potential velocities allowed on the contact.

Kinematic tasks can be defined for the following situations:

- Tasks in which the fingertips are compatible with the object geometry and the object motion, without changing the position of the fingertips. The contact point does not change.
- Tasks of rolling on the object surface, for a moving object.
- Tasks of sliding on the object surface, for a moving object.

In all these situations, force equilibrium can be enforced. This work focuses on the first two points for particular geometries.

Assume that the geometry of the convex object is known and has principal curvatures ρ_{ai} and ρ_{bi} at the contact point with finger i, P_i. Each point has a corresponding *surface frame*, with the local z axis is directed towards the object and the local x and y axes at the plane tangent to the surface and correspond to the directions of maximum and minimum curvature, and so that they form a direct trihedron.

Consider the fingertip as the center of a sphere of radius r, for a hand with b fingers. At the reference configuration, the local frames of the fingers are translated in the negative z direction from their corresponding surface frames, with the origin of these local frames located at the center of the sphere corresponding to each fingertip, see Figure 3.

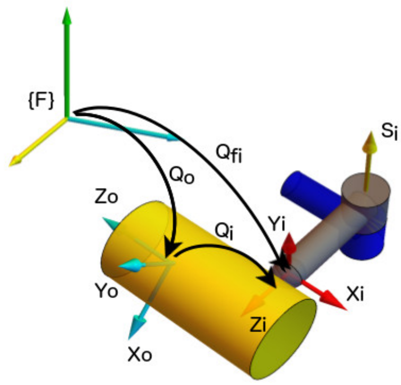

FIGURE 3. Transformation to finger frame.

An arbitrarily-located moving frame is attached to the object to be manipulated. The contact points $P_1,\ldots,P_b$ on the object can be calculated using a grasp synthesis method and are given as local displacements from the object frame to the surface frame, Q_i, with $i=1,\ldots,b$. The end-effector for each finger i must be then located as

$$Q_{fi} = Q_o Q_i (1 - \frac{r_i}{2} k\varepsilon), \quad i = 1,\ldots,b, \tag{21}$$

where Q_o is the known position of the object.

4.1. **Motion of the object with fixed contact points.** In this simplest case of motion, the transformations $Q_1,\dots,Q_b$ are fixed; no rolling, sliding or losing of contact is allowed for the fingers. If nonfiction is assumed, then the rotation about the z axis, $Q_{Rz} = \cos\frac{\phi}{2} + \sin\frac{\phi}{2}z$, is allowed.

For an m-position synthesis task, with $m = m_p + m_v$ for finite displacement and velocities, define m_p positions of the object, $Q_o^1,\dots,Q_o^m$, the simultaneous task for each finger is

$$Q_{fi}^j = Q_o^j Q_i (1 - \frac{r_i}{2} k\varepsilon), \quad j = 1,\dots,m_p, \quad i = 1,\dots,b. \tag{22}$$

Usually the synthesis uses relative positions with respect to a reference configuration, taken as the first position. In this case, the relative position for each finger is identical,

$$Q_{fi}^{1j} = Q_{fi}^j (Q_{fi}^1)^* = Q_o^j (Q_o^1)^* = Q_o^{1j}, \quad j = 2,\dots,m_p, \quad i = 1,\dots,b, \tag{23}$$

that is, considering the fingers as rigidly attached to the object. This is similar to the synthesis problem for parallel robots, in which each of the legs of the robot has to reach the relative motion of the platform. The finite-position synthesis is possible in this case if the mobility of the hand-object system is positive.

A task with object velocities can be transformed to a task with positions or positions and velocities at the fingertips. Consider the object twist V_o^j associated to the object displacement Q_o^j. This fully defines the velocities of the fingertips, as

$$V_{fi}^j = V_o^j, \quad j = 1,\dots,m_v, \quad i = 1,\dots,b. \tag{24}$$

The velocity of the origin of the finger frame can be easily calculated from the object twist and the displacement as

$$v_i = v_0 + w \times p_i. \tag{25}$$

4.2. **Motion of the object with sliding fingers.** In this case, the transformations $Q_1^j,\dots,Q_b^j$ are to be specified for each position j in such a way that they are compatible with the geometry of the object. The sliding allows to keep the same contact point in the spherical finger, so that the transformation from the surface to the finger frame is constant, $(1 - \frac{r_i}{2} k\varepsilon)$.

For an m-position synthesis task, define again m_p positions of the object, $Q_o^1,\dots,Q_o^m$, the simultaneous task for each finger is

$$Q_{fi}^j = Q_o^j Q_i^j (1 - \frac{r_i}{2} k\varepsilon), \quad j = 1,\dots,m_p, \quad i = 1,\dots,b. \tag{26}$$

The relative position with respect to the first position for each finger is

$$Q_{fi}^{1j} = Q_o^j Q_i^j (Q_i^1)^* (Q_o^1)^*, \quad j = 2,\dots,m_p, \quad i = 1,\dots,b. \tag{27}$$

The selection of the displacements Q_i can follow a surface trajectory. Assume that the initial points obtained in the grasp planning stage are Q_i^1. Global trajectories can be calculated if the geometry is well know. A local approximation from the first position can be calculated using Taylor's series, with

$$Q_i(t) = Q_i^1 + \frac{dQ_i}{dt}\rfloor_1 t + \ldots, \tag{28}$$

so that the trajectory of a point on the surface is given by

$$\begin{aligned} X(t) &= (Q_i^1 + \frac{dQ_i}{dt}\rfloor_1 t + \ldots)(Q_i^1)^* X Q_i^1((Q_i^1)^* + (\frac{dQ_i}{dt}\rfloor_1 t)^* + \ldots) \\ &= (1 + \frac{1}{2}V_1 t + \ldots)X(1 + \frac{1}{2}V_1^* t + \ldots), \end{aligned} \tag{29}$$

where V_1 must be such that the point velocity is tangent to the surface, that is, in the local $x-y$ plane. For curved surfaces, it is necessary to incorporate the second derivative in order to move along the surface. For this work we focus on convex polygonal objects.

In the surface frame, the relative twist of the finger is

$$V_{ri} = (v_x i + v_y j)\varepsilon \tag{30}$$

so that the overall finger twist can be linearly calculated,

$$\begin{aligned} V_{fi}^j = V_i^j = V_o^j + Q_{fi}^j V_{ri}^j Q_{fi}^{j\,*}, \quad & j = 1, \ldots, m_v, \\ & i = 1, \ldots, b. \end{aligned} \tag{31}$$

For a synthesis task, more than one twist can be defined at a given position, effectively defining the subspace of potential velocities of the fingertip. Given a hand with mobility $M > 0$ and $M' = 0$, defining M velocities for the fingertips fully specifies the allowable twists and ensures that the fingertip's motion will be in the desired subspace for each position.

For the case of polygonal objects,

$$\begin{aligned} V_{ri_1} &= v_x i\varepsilon \\ V_{ri_2} &= v_y i\varepsilon \end{aligned} \tag{32}$$

defines the sliding on the surface of the object for a particular position. Applying these conditions, the fingertips of the synthesized hand will move on the surface of the object while performing the specified motion of the object.

5. Conclusions

The design of hands for specific grasping and manipulation tasks using kinematic synthesis requires a definition of fingertip displacements and velocities compatible with the object geometry, selected contact points, desired object motion and type of contact fingers. In this work a first analysis is developed in order to create kinematic tasks of fingers to create hands able to manipulate objects while keeping some grasping constraints. The initial cases presented here need to be developed to include other type of finger actions and link contacts. The use of these tasks may lead to the design of hands better tailored to specific applications.

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