

Review: Adventures among the toroids, by Bonnie Stewart

by Henry Crapo

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Résumé

La remarquable monographie intitulée «*Aventures parmi les toroïdes*» de Bonnie Stewart a été récemment republiée après dix ans d'absence.

Le but de cet ouvrage est identifié dans sa page-titre au style victorien. «*Aventures ...*» est «une étude des polyèdres quasi-convexes, non planaires, à tunnels orientables, de genre positif ayant des faces régulières à intérieurs séparés, étant une description élaborée et donnant des instructions pour la construction d'un nombre important et fascinant de modèles mathématiques qui sont d'un grand intérêt pour les étudiants en géométrie et topologie euclidiennes, qu'ils soient de niveau secondaire ou collégial, pour les designers, les ingénieurs et architectes, pour un public de scientifiques impliqués dans des problèmes moléculaires ou autres problèmes structuraux, et pour les mathématiciens, professionnels ou non, comportant des centaines d'exercices et de projets de recherche, dont beaucoup ont été prévus pour permettre l'étude personnelle.»

Recently republished after a ten-year interval, Bonnie Stewart's remarkable monograph «*Adventures among the toroids*» is now back in circulation. The text of this revised second edition is again hand-lettered in a flawless chancery script and profusely illustrated, both by the author.

The purpose of the book is stated in its Victorian-style title page. «*Adventures...*» is «a study of Quasi-Convex, aplanar, tunneled orientable polyhedra of positive genus having regular faces with disjoint interiors, being an elaborate description and instructions for the construction of an enormous number of new and fascinating mathematical models of interest to students of euclidean geometry and topology, both secondary and collegiate, to designers, engineers and architects, to the scientific audience concerned with molecular and other structural problems, and to mathematicians, both professional and dilettante, with hundreds of exercises and search projects, many outlined for self-instruction.»

This new edition is highly recommended for those interested in the juxtaposition of polyhedra, and of regular and regular-faced polyhedra in particular. The author's approach is systematic and pains-taking throughout. Her writing style is invariably clear, pleasant and informative. We illustrate the contents of «*Adventures...*» by reprinting as **Figure 1** its illustration of Norman Johnson's «oriental hat», and as **Figure 2** a page of discussion of bilunabirotunda». **Figure 3** shows how six of these polyhedra can be clustered around a cube. **Figure 4** shows the construction of a zig-zag tunnel through the bilunabirotunda, a construc-

tion which the author states «astounded me, and led to sextillions of new models». **Figure 5** (p 152) gives an idea of the style of writing.

Copies are available by writing directly to the author, enclosing a payment of US \$11 (with perhaps some allowance for inflation and postage): B.M. Stewart, 4949 Wausaw Road, Okemos, Michigan 48864.

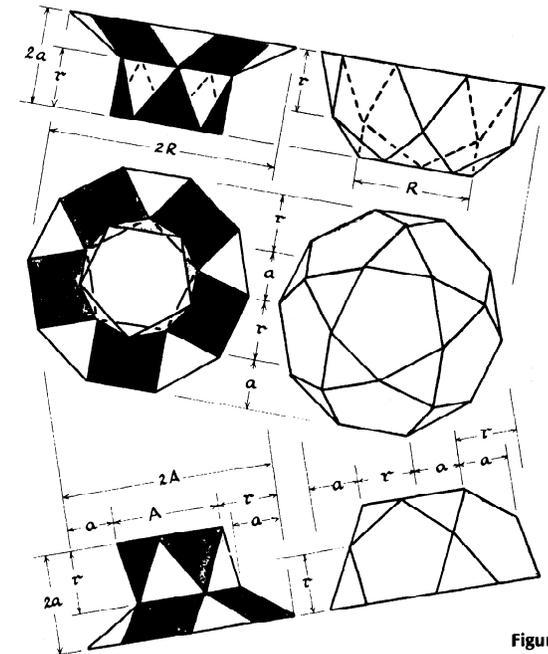
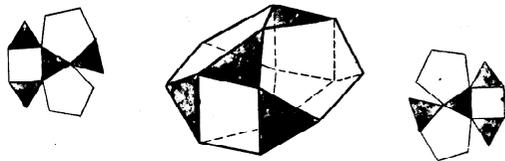


Figure 1

The Bilunabirotunda

$$L_2^2 R_2^2 = J_{91}, \quad p=0$$



$$F=14, V=14, E=26$$

One of the convex polyhedra with regular faces first described by Dr. Johnson is the "bilunabirotunda" denoted by $L_2^2 R_2^2$. Since this is No. 91 in his paper we use the shorter notation J_{91} . To obtain a view showing the upper 5:3:5:3 or R_2 portion and its relation to the lower similar portion - accounting for the "birotunda" in the name and the R_2^2 in the notation, we may superimpose the ground plan of the pentagonal rotunda on the drawing which shows the relations of the pentagon with unit edge and the decagon with unit edge. In this view one pair of parallel pentagonal faces of J_{91} is shown in exact size. Of course these faces are offset, but the center C of one is directly above a vertex U' of the other. Thus the distance between these parallel pentagonal faces is the same as the altitude of the rotunda:

$$h(5,5, J_{91}) \approx 2a$$

Figure 2

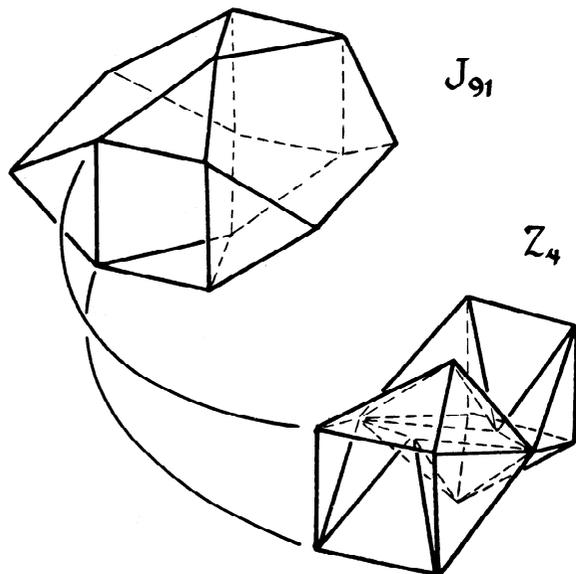


Figure 4a

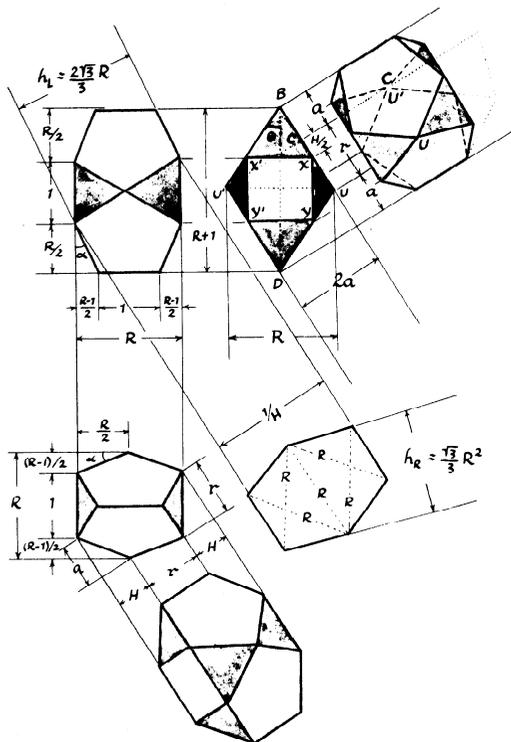


Figure 3

A Zig-Zag tunnel through the Bilunabirotunda

Z_4 is a regular-faced non-convex polyhedron with $p=0$, with two square faces, like those in J_{91} , separated by a distance R , and with the other twenty faces triangular. For Z_4 we have $F=22, V=14, E=34$. If the square ends of Z_4 are left open, the side view shows the zig-zag "Z"-shape flow of the interior space which suggested the letter name Z_4 .

Since $\tan \beta \approx (3R-4)/(R+7) < (R-1)/R \approx \tan \alpha$, we may use Z_4 as a tunnel in J_{91} , by rotating Z_4 through 90° with respect to the parent figure.

$$J_{91}/Z_4, \quad p=1, \quad F=32, \quad V=20, \quad E=52.$$



In the sequel we shall make several exciting applications of Z_4 , taking advantage of its comparatively small girth and the important altitude property:

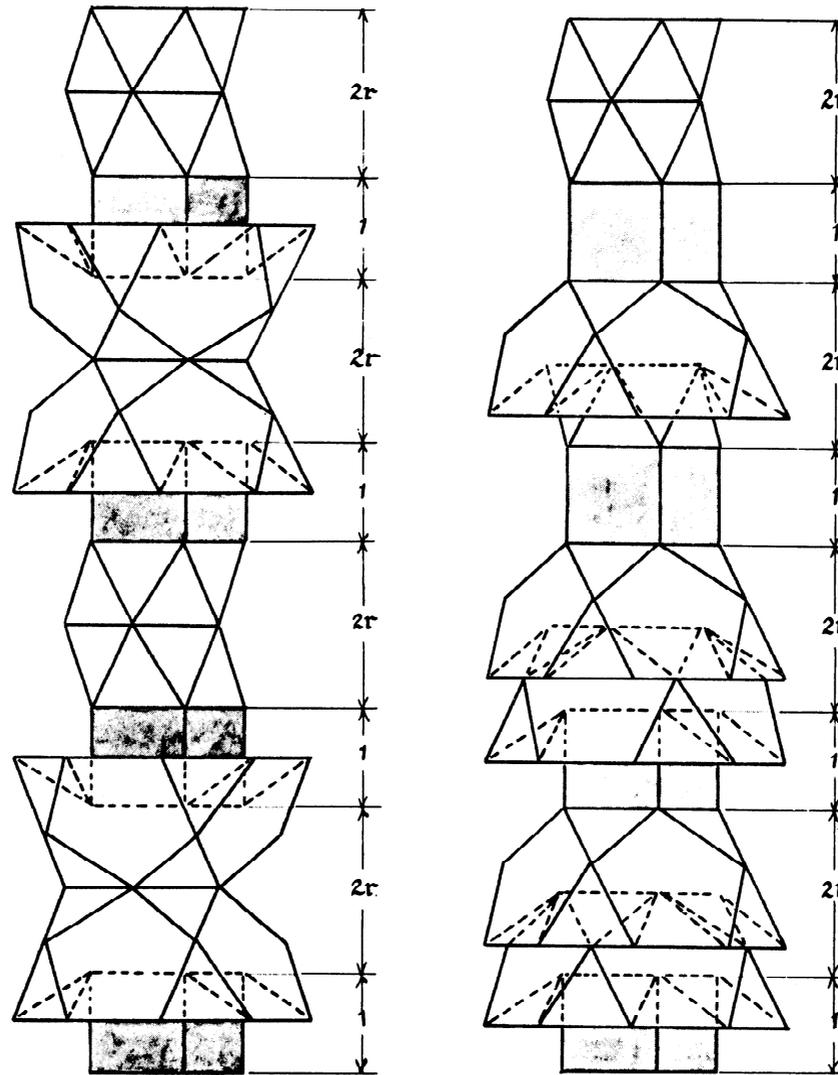
$$h(4,4, Z_4) \approx R$$

Figure 4b

Our stay in the land of the toroids was near the end of its first year when we first learned of the clan or cult of the "Double-Ess-Five". Since we subsequently earned our way to a high place in that organization, perhaps we can be forgiven a somewhat lengthy, albeit less than scientific, account of that adventure.

In our wanderings we became accustomed to the multi-tunneled structures that seemed natural dwellings and places of commerce or entertainment for a people who lived among and dreamed only of spheres with handles. Imagine our astonishment when we first saw at a distance a restricted area filled with pentagonal towers, some of great height - and some leaning badly like Pisa, but not a tunnel in sight, so at first we thought this to be some ancient burial place, built before the delights of the neotroglodyte, holey life had been realized.

On closer inspection we found the towers were more like totem poles or stylized xats, regular-faced and planar, but not quasi-convex. Of



$S_5 S_5$ P_5 $Q_5^* R_5 R_5 Q_5^*$ P_5
 $S_5 S_5$ P_5 $Q_5^* g(R_5 R_5) Q_5^*$ P_5

genus $p=0$, each rose intlers with pentagonal prisms P_5 in alternate positions, each separated from the next by structures which we soon saw had a common height which on measurement proved to be $h(5,5, S_5 S_5) \approx 2r$.

So we could find the total height as a multiple of $1+2r$ by merely counting the number of tiers, for the poles were invariably started with P_5 and crowned with $S_5 S_5$.

Our inquiries confirmed these observations. The intervening units were much varied - and the makers of the poles, who were not ancients, but quite modern, were held in great esteem according to the novelty of the structures they invented. In fact they were banded into a secret order, called the "Double-Ess-Five", after their continuing search for every (R)(A) polyhedron P with the property $h(5,5, P) \approx 2r \approx h(5,5, S_5 S_5)$.

Perhaps our on-the-spot sketch of one of the towers will convey the flavor of the fraternity's enterprise. With our usual abbreviations this tower may be described, from the top, as

$S_5 S_5$ P_5 $R_5 Q_5^* S_5$ P_5
 $R_5 Q_5^* R_5 Q_5^*$ P_5 $R_5 g(Q_5^* R_5) Q_5^*$ P_5

Figure 5

This is a good time to point out that several constructions reported by Koji Miyazaki and Ichiro Takada (*Structural Topology #4*) were also discovered by Bonnie Stewart and her student Kurt Schmucker. We point this out in order to emphasize once more the advantage of establishing and maintaining contact between those doing research on structural and morphological problems. With the publication of *Structural Topology*, an exchange of problems, new constructions and results is sure to be accelerated.

For a case in point, we find Ichiro Takada's model, Figure 21(c), with central octahedron in place, illustrated on page 52 of the new edition of *«Adventures ...»*. We reprint Bonnie Stewart's drawing as **Figure 6**. Schmucker and Stewart likewise constructed the ring of eight regular dodecahedra and joined these to form certain of the «golden isozonohedra», such as Koji Miyazaki's Figure 26(1) or (2), as illustrated on page 60 of *«Adventures ...»* (our **Figure 7**). Stewart also applied these methods to the construction of knotted toroids (page 216-217 of *«Adventures ...»*).

New construction techniques permitted Miyazaki and Takada to achieve models beyond the reach of the scissors and paste approach. Thus, the construction of the equivalent of Miyazaki's Figure 26(3) evokes the following amusing paragraph in *«Adventures ...»*:

«Believe it, Schmucker actually constructed some such models. He charmed a bevy of secretaries into lunch-hour cutting of pentagonal faces, floated a loan for paper and rubber-bands and photographic records of progressive steps in the construction — and persuaded a janitor to find an otherwise unused storage room for keeping the growing structure out of reach of curious or irate roommates ...»

In closing, we note that Bonnie Stewart also has a home-made «Poly-kit» on pages 20-21, each piece prepared with scissors, a straight edge and a dry ball-point pen.

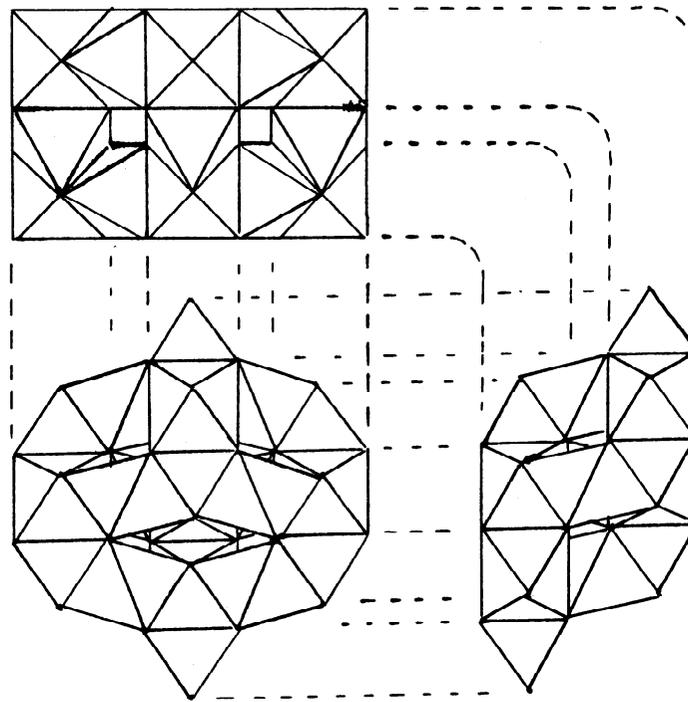


Figure 6

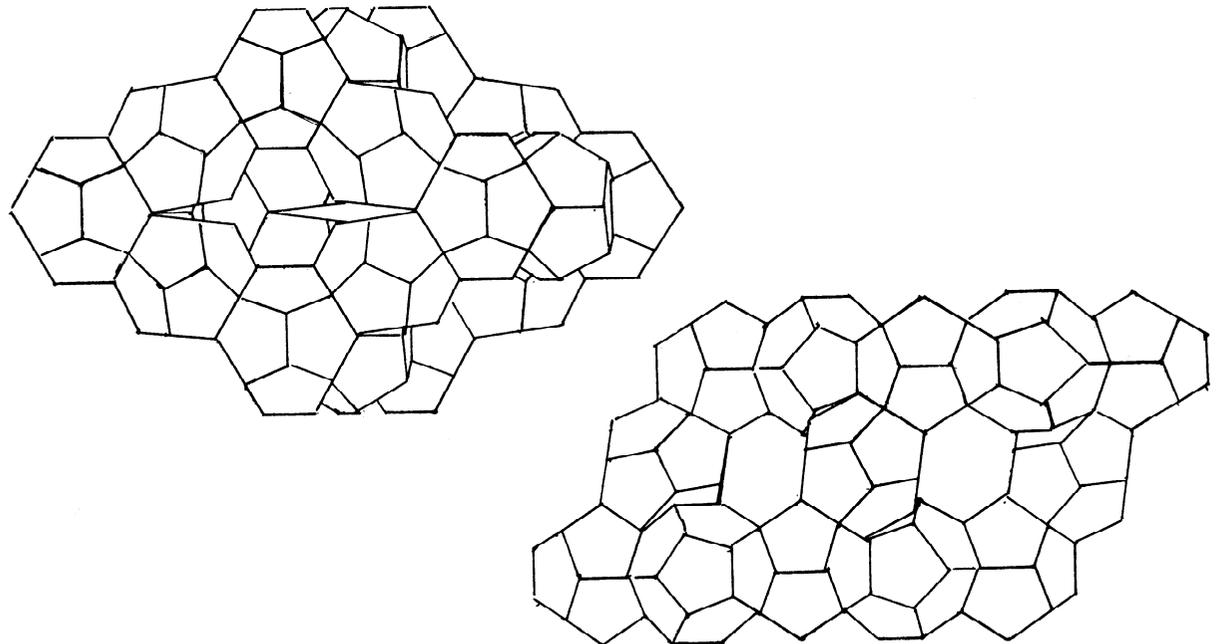


Figure 7