Collocation Methods for 2nd Order Systems

S. Moreno–Martín, L. Ros, and E. Celaya

IRI (UPC – CSIC), Barcelona

Usual methods

(for 1st order systems)

Trapezoidal $\quad x_{k+1} = x_k + \frac{h}{2}(f_{k+1} + f_k)$

Hermite Simpson $\quad x_{k+1} = x_k + \frac{h}{6}(f_k + 4f_c + f_{k+1})$

Meant for 1st order systems

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t),$$

but in robotics we have

$$\ddot{\mathbf{q}} = \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{u}, t)$$

Usual workaround:

Define → $\mathbf{x} = (\mathbf{q}, \mathbf{v})$

Add → $\mathbf{v} = \dot{\mathbf{q}}$

to convert to 1st order form

$$\begin{cases} \dot{\mathbf{q}} = \mathbf{v} \\ \dot{\mathbf{v}} = \mathbf{g}(\mathbf{q}, \mathbf{v}, \mathbf{u}, t) \end{cases}$$

But since

$\mathbf{q}(t)$, $\mathbf{v}(t)$ are approximated by polynomials of the same degree,

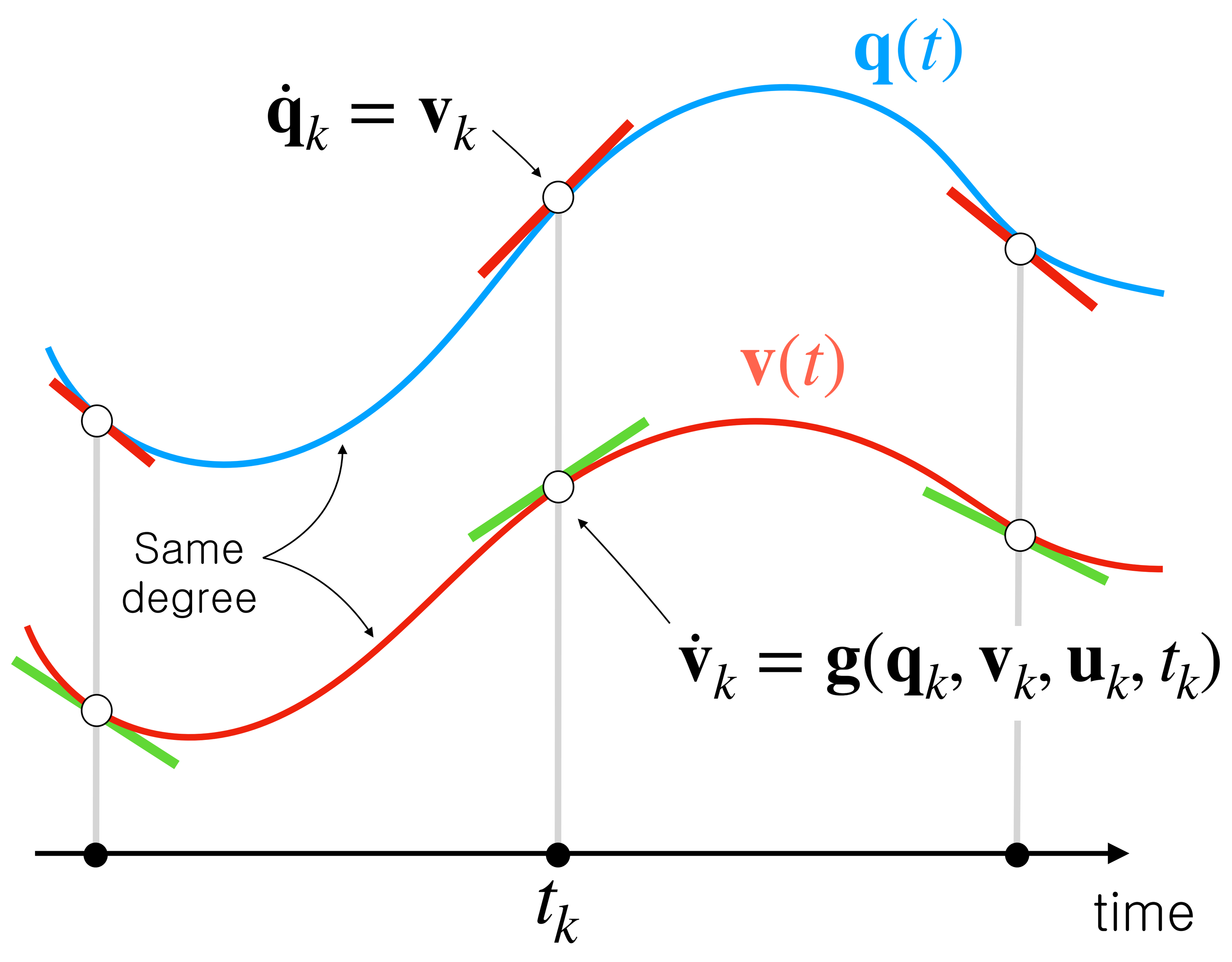
in these polynomials it will be

$\dot{\mathbf{q}}(t) \neq \mathbf{v}(t)$
(except at coll. points)

$\ddot{\mathbf{q}}(t) \neq \dot{\mathbf{v}}(t)$
(even at coll. points)

yields

Inconsistency in usual collocation



Since

$$\ddot{\mathbf{q}}_k \neq \dot{\mathbf{v}}_k$$

then

$$\dot{\mathbf{v}}_k = \mathbf{g}(\mathbf{q}_k, \mathbf{v}_k, \mathbf{u}_k, t_k)$$

$$\Downarrow$$

$$\ddot{\mathbf{q}}_k \neq \mathbf{g}(\mathbf{q}_k, \dot{\mathbf{q}}_k, \mathbf{u}_k, t_k)$$

so the 2nd order dynamics is not satisfied

Increases dynamic error, so extra control effort is needed to track

$$\mathbf{x}(t) = \big(\mathbf{q}(t), \mathbf{v}(t)\big)$$

New methods

(for 2nd order systems)

Trapezoidal
$$q_{k+1} = q_k + v_k h + \frac{h^2}{6}(g_{k+1} + 2g_k)$$
$$v_{k+1} = v_k + \frac{h}{2}(g_{k+1} + g_k)$$

Hermite Simpson
$$q_{k+1} = q_k + v_k h + \frac{h^2}{6}(g_k + 2g_c)$$
$$v_{k+1} = v_k + \frac{h}{6}(g_k + 4g_c + g_{k+1})$$

Advantages

Guarantee $\quad \dot{\mathbf{q}}(t) = \mathbf{v}(t) \quad \forall t$

Impose actual 2nd order dynamics
$$\ddot{\mathbf{q}} = \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{u}, t)$$
at the collocation points

Reduce dynamic error in more than one order of magnitude

Do not increase the computation time significantly

Trajectories will be tracked with less control effort

Yield twice differentiable trajectories