A precision grasp on an object:
- Stable grasp
- Finger-to-point assignment
- Normal on contact point

Find all configurations of the hand-object system
Multiple configurations are possible, even infinite

Problem statement

Formulation

The variables are in global coordinates
Algebraic equations with linear, bilinear and quadratic monomials

I. Loop constraints

II. Finger kinematic and reference frame constraints

III. The contact point is fixed and modelled as a revolute joint

IV. Joint angles are limited by constraining their sines and cosines, defined by dot and cross products of finger vectors

Dimension of the solution

For \( n \) fingers:

- \( f = 5n \) degrees of freedom
- \( c = 6(n - 1) \) constraints
By the Grübler-Kutzbach’s criterion

\[ d = f - c = 6 - n \]

Numerical Solution

Branch-and-prune algorithm based on linear relaxations

Pre-processing step

\[ x_i, x_i^2 \quad \text{and} \quad \begin{align*}
L(x) &= 0, \\
Q(x) &= 0, \\
B(x) &= 0.
\end{align*} \]

initial system -> change of variables -> new system

Iteration of 2 basic steps

Shrink box
Linear programming problem

Split box
Trivial bisection

Test cases

Snapshots of 1-dimensional solution subset