Real-Time Obstacle Avoidance for trailer-like Systems

T.A. Vidal-Calleja†, M. Velasco-Villa†, E. Aranda-Briceaire‡†
†Departamento de Ingeniería Eléctrica, Sección de Mecatrónica, CINVESTAV-IPN, A.P. 14-740, 07000, México D.F.
{tvidal,velasco}@mail.cinvestav.mx
‡Programa de Matematicas Aplicadas y Computación, Instituto Mexicano del Petroleo, A.P. 14-805, 07730, México D.F.
earanda@mail.cinvestav.mx

Abstract

The obstacle avoidance problem associated to a multi-steered general 1-trailer is studied in this work by considering the method of artificial potential fields. It is proposed a control strategy based on artificial potential fields that generate a trajectory to be followed by the tractor that represents, at the same time, a reference for the trailer. The control laws proposed in this paper are experimentally implemented on a prototype built at the laboratory.

1 Introduction

A trailer-like system is a mobile robot composed by a tractor which pulls n trailers connected by a revolute joint. There are two main models of trailer systems considered in the literature. Namely, the standard n-trailer and general n-trailer. The difference between these two models arises from the position of the joint. The joint of the standard n-trailer is located at the rear axle, while the joint of the general n-trailer is located off this axle. The artificial potential field method was developed in [5, 6], and has been extensively studied in the obstacle avoidance problem for autonomous mobile robots [1, 3, 8]. There are two main approaches for this method. On one hand, the classical method [5] depends only on the position of the mobile robot. On the other hand, the generalized potential field method [6] depends also on the velocity of the robot. The underlying idea of the method is to fill the robot’s workspace with an artificial potential field in which the vehicle is attracted to its goal and is repulsed away from the obstacles.

2 Kinematic Model

A multi-steered general 1-trailer composed by a mobile robot and a trailer with a steered wheel is considered. The position of the center of the tractor axle with respect to the fixed reference frame $(X_1, X_2)$ is given by $(x_1, x_2)$. The orientation $\theta_0$ corresponds to the angle that the robot forms with respect to $X_1$, as shown in Figure 1. The orientation of the trailer is denoted by $\theta_1$. The direction of the actuated wheels with respect to the longitudinal axis of the trailer is denoted $\beta_1$. The variables $u_1, u_2$ represent respectively the linear and angular velocities of the tractor and $u_3$ represents the angular velocity of the steering wheel for the trailer. The position of the center of the tractor axle is given by $(w_1, w_2)$. The distance between the point $(x_1, x_2)$ and the joint is given by $d_0$ and the distance between this joint and the point $(w_1, w_2)$ is denoted $d_1$. With this notation the
kinematic model is given by [7],

\[
\begin{align*}
\dot{x}_1 &= u_1 \cos \theta_0 \\
\dot{x}_2 &= u_1 \sin \theta_0 \\
\dot{\theta}_0 &= u_2 \\
\dot{\theta}_1 &= a_1 (\theta_0, \theta_1, \beta) u_1 - a_2 (\theta_0, \theta_1, \beta) u_2 \\
\dot{\beta}_1 &= u_3,
\end{align*}
\]

where,

\[
\begin{align*}
a_1 &= \frac{1}{d_1 \cos \beta_1} \sin(\theta_0 - \theta_1 - \beta_1) \\
a_2 &= \frac{d_0}{d_1 \cos \beta_1} \cos(\theta_0 - \theta_1 - \beta_1).
\end{align*}
\]

3 Artificial Potential Field Method

A gradient system [4] on an open set \( W \subset \mathbb{R}^n \) is a dynamic system of the form

\[
\dot{x} = -\nabla V(x)
\]

where \( V: U \to \mathbb{R} \) is a \( \mathcal{C}^2 \) function, and

\[
\nabla V = \left( \frac{\partial V}{\partial x_1}, \ldots, \frac{\partial V}{\partial x_n} \right)
\]

is the gradient vector field, \( \nabla V: U \to \mathbb{R}^n \) of \( V \).

Regular points of the trajectories of (2), cross level surfaces of the function \( V(x) \) orthogonally.

Denote the position of a point of the vehicle in a two-dimensional workspace by \( q = (y_1, y_2) \), the position of the goal by \( q_g = (y_{1g}, y_{2g}) \) and the position of a unique obstacle by \( q_o = (s_1, s_2) \). An artificial potential function applied to the vehicle at point \( q \), has the form,

\[
U(q) = U_a(q) + U_r(q),
\]

where \( U_a(q) \) is the attractive potential induced by the goal and \( U_r(q) \) is the repulsive potential induced by the obstacle. The resultant force is then obtained as,

\[
F = F_a + F_r,
\]

where,

\[
\begin{align*}
F_a(q) &= -\nabla U_a(q), \\
F_r(q) &= -\nabla U_r(q).
\end{align*}
\]

In the sequel, \( \| \cdot \|: \mathbb{R}^2 \to \mathbb{R}^+ \) denotes the usual Euclidean distance function.

The attractive force \( F_a \) guides the robot to the goal and \( F_r \) is a repulsive force which repels the vehicle from the obstacle. Usually the attractive potential is defined by,

\[
U_a(q) = \frac{1}{2} \xi \rho(q, q_g),
\]

where \( \xi \) is a positive scaling factor and \( \rho(q, q_g) = \| q - q_g \|^2 \) is a positive definite function whose first derivative is continuous and has a global minimum equal to zero at \( q = q_g \).

The repulsive potential function takes the form,

\[
U_r(q) = \begin{cases} 
\frac{1}{2} \eta \left( \frac{1}{\rho(q, q_o)} - \frac{1}{\rho_0^2} \right)^2 & \text{if } \rho \leq \rho_0^2 \\
0 & \text{if } \rho > \rho_0^2,
\end{cases}
\]

where \( \eta \) is a positive scaling factor and \( \rho(q, q_o) \) is defined above. The region of influence of the obstacle is determined by the positive constant \( \rho_0 \). Notice that \( U_r(\cdot) \) is once continuously differentiable, e.g. \( U_r(q) \in \mathcal{C}^1(\mathbb{R}^2) \).

From equations (5, 6, 7), the induced forces can be expressed as,

\[
\begin{align*}
F_a(q) &= -\xi (q - q_g) \\
F_r(q) &= \eta \left( \frac{1}{\rho(q, q_o)} - \frac{1}{\rho_0^2} \right) \times \\
& \quad \frac{\partial \rho(q, q_o)}{\partial q} \\
& \quad \frac{\partial \rho(q, q_o)}{\partial q_o^i} + \frac{\partial \rho(q, q_o)}{\partial q_o^j}
\end{align*}
\]

where \( F_r(q) \) is applied if \( \rho \leq \rho_0 \) and is set to zero otherwise.

In the case of \( n \) obstacles the above procedure can be generalized by considering the force \( F_r \) associated to \( i \)-th obstacle, producing

\[
F_r = \sum_{i=1}^{n} F_{r_i}.
\]

Figure 2 shows the potential function for a three obstacles case. It is easy to see that the goal is a global minimum and the potential function indefinitely grows inside the region of influence of the obstacles.
The coordinates of this point are given by,
\( c \) where the time-derivative of (9), produces,
\[
\begin{bmatrix}
\dot{y}_1 \\
\dot{y}_2
\end{bmatrix} = A(\theta_0) \begin{bmatrix}
u_1 \\
u_2
\end{bmatrix},
\]
(10)
with
\[
A(\theta_0) = \begin{bmatrix}
\cos \theta_0 & -\ell \sin \theta_0 \\
\sin \theta_0 & \ell \cos \theta_0
\end{bmatrix}.
\]

Consider the desired values \( \dot{q}_d = (\dot{y}_{1d}, \dot{y}_{2d}) \) to be proportional to the normalized force generated by the potential field [2],
\[
\begin{bmatrix}
\dot{y}_{1d} \\
\dot{y}_{2d}
\end{bmatrix} = \frac{v_d}{\sqrt{f_1^2 + f_2^2}} \begin{bmatrix}
f_1 \\
f_2
\end{bmatrix},
\]
(11)
where \( f_1, f_2 \) are respectively the components of the total force \( F \) in the direction of the axes \( X_1 \) and \( X_2 \) and \( v_d \) is the desired scalar velocity. Under this conditions, by considering equations (10) and (11) it is possible to propose the feedback control law:
\[
\begin{bmatrix}
u_1 \\
u_2
\end{bmatrix} = \frac{v_d}{\sqrt{f_1^2 + f_2^2}} A^{-1}(\theta_0) \begin{bmatrix}
f_1 \\
f_2
\end{bmatrix}.
\]
(12)

Remark 1 Note that the classical method depends only on the relative position of the obstacles and vehicle. Notice that also the vehicle moves in the force field direction.

4 Control Strategy

It is intended to take the vehicle from initial to a final position without colliding with obstacles. The control strategy will be developed in two steps. First, the tractor control law will be designed using an artificial potential field. Then the tracking problem of the trailer will be analyzed. By using the potential field method the tractor will follow the direction of the resultant force. The trajectory generated by the tractor is considered as a reference trajectory for the trailer.

4.1 Tractor Control

Consider the output function \( q = (y_1, y_2) \) as the point that represents the middle of the front end of the tractor. The coordinates of this point are given by,
\[
y_1 = x_1 + \ell \cos \theta_0 \\
y_2 = x_2 + \ell \sin \theta_0,
\]
(9)
where \( \ell \) is the orthogonal distance between the rear axle point \( (x_1, x_2) \) and the point \( q \) of the robot. Taking the time-derivative of (9), produces,
\[
\begin{bmatrix}
\dot{y}_1 \\
\dot{y}_2
\end{bmatrix} = A(\theta_0) \begin{bmatrix}
u_1 \\
u_2
\end{bmatrix},
\]
(10)
with
\[
A(\theta_0) = \begin{bmatrix}
\cos \theta_0 & -\ell \sin \theta_0 \\
\sin \theta_0 & \ell \cos \theta_0
\end{bmatrix}.
\]

On the other hand, the repulsive potential and the components of its negative gradient, inside of the region of influence, are given by,
\[
U_r = \frac{\eta}{2} \left( \frac{1}{(y_1 - s_1)^2 + (y_2 - s_2)^2} - \frac{1}{\rho_0} \right)^2
\]
\[
f_{1r} = 2\eta \left( \frac{1}{(y_1 - s_1)^2 + (y_2 - s_2)^2} - \frac{1}{\rho_0} \right) \times \frac{y_1 - s_1}{((y_1 - s_1)^2 + (y_2 - s_2)^2)^2}
\]
\[
f_{2r} = 2\eta \left( \frac{1}{(y_1 - s_1)^2 + (y_2 - s_2)^2} - \frac{1}{\rho_0} \right) \times \frac{y_2 - s_2}{((y_1 - s_1)^2 + (y_2 - s_2)^2)^2}.
\]

4.2 Trailer Control

As mentioned before, it is intended that the trailer of the vehicle follows the path described by the tractor as a result of the artificial potential field applied to it. One way to achieve the above goal is to apply a control strategy that makes the center of the trailer axle \( (w_1, w_2) \) to follow approximately the position of a point in the tractor \( (y_1, y_2) \), subject to a given time delay \( \tau \). This is, a point in the trailer has to take the same position that was generated by the tractor. To implement this strategy, consider \( \beta \) as the output of the trailer subsystem. The angle \( \beta \) will be manipulated to indirectly control the position of a point in the trailer.

Consider the error \( e_\phi \) defined as,
\[
e_\phi = (\phi - \phi_d),
\]
where $\phi = \arctan\left(\frac{w_2-y_2(0)}{w_1-y_1(0)}\right)$ is the angle of point $(w_1, w_2)$ with respect to the initial position of the tractor and $\phi_d$ corresponds to its desired value. Note that the sign of the variable $e_{\phi}$ indicates in which side of the desired trajectory the trailer is placed.

In order to take into consideration the path generated by the potential field over the tractor (point $(y_1, y_2)$), the desired value $\phi_d$ it is proposed as,

$$\phi_d = \arctan\left(\frac{y_2(t-\tau) - y_2(0)}{y_1(t-\tau) - y_1(0)}\right),$$

where the time delay $\tau$ is given by,

$$\tau = \frac{d_1 + d_2}{v_d}. \quad (13)$$

Remark 2 Note that due to the normalization of the forces generated by the potential field $(11)$, the scalar velocity of the point $(y_1, y_2)$ is constant.

Now, it is possible to define a desired value for the output $\beta$, of the trailer subsystem, in terms of the relative position of the tractor and trailer by,

$$\beta_d = -ke_{\phi},$$

from what, a control feedback law for the trailer is given by,

$$u_3 = -\alpha \tanh(\gamma (\beta - \beta_d)), \quad (14)$$

where $\alpha, \gamma$ are positive constants.

Figure 3 shows a block diagram of the complete control scheme (Tractor-trailer).

5 Real-Time Experiments

To evaluate the obstacle avoidance control strategy developed before a real-time implementation is made. The prototype was designed at the Mobile Robots Laboratory of the Mechatronics Section of the Electrical Engineering Department at CINVESTAV-IPN. Figure 4 shows a picture of the trailer system prototype.

Due to the lack of sensors on the prototype it is necessary to assume the knowledge of the goal and obstacles position. The obstacles are located at

$q_{o(1)} = (1, 0), q_{o(2)} = (2, -0.5), q_{o(3)} = (2, 1)$

and the goal was set at $q_g = (5, 0)$.

The initial conditions of the vehicle was taken as $x_1(0) = 1, x_2(0) = -0.5, \theta_0(0) = 0.78, \theta_1(0) = 0.78, \beta(0) = 0.$

The position of the vehicle is estimated by the kinematic model, the angular position of the motors of the wheels of the tractor are sensed by incremental encoders. The angle between tractor and trailer and the angular position of the motor on the axle of the trailer are sensed by linear potentiometers.

The relationship between the linear and angular velocities of the tractor and the angular velocities of each wheel are given by,

$$\begin{pmatrix} \omega_d \\ \omega_i \end{pmatrix} = T \begin{pmatrix} u_1 \\ u_2 \end{pmatrix},$$

with

$$T = \begin{pmatrix} \frac{r}{L} & \frac{r}{L} \\ \frac{r}{d} & \frac{r}{d} \end{pmatrix},$$

where $r$ is the radius of the wheels and $L$ is the length of the rear axle of the tractor.

The parameters of the experimental prototype are,

$$d_0 = 0.48, d_1 = 0.38, \ell = 0.3, L = 0.144, r = 0.065.$$
The trajectories followed by the tractor and the trailer are shown in Figure 5, where it is possible to see how the path of the tractor represent, as expected, a reference for a trailer. In Figure 6, the control $u_1$, $u_2$, $u_3$, applied to the vehicle are depicted. The velocity signals used to control the vehicle are transformed to voltages by using a classical PID internal loop. This is possible by assuming that the electrical dynamics is faster than the mechanical one. The voltages supplied to the motors are presented in Figure 7. In Figure 8 is shown the error $\beta - \beta_d$.

6 Conclusions

The obstacle avoidance problem associated to a trailer-like vehicle has been analyzed in this work. The study is done by considering a multi-steered general 1-trailer,
a general model that presents a steered wheel for the trailer. It is proposed a control strategy based on artificial potential fields that generate a trajectory that is followed by the tractor. This trajectory is considered as a reference for the control applied to the trailer. The performance of the control strategy was tested by real-time experiments.

References


