ARTIFICIAL POTENTIAL FIELDS FOR TRAILER-LIKE SYSTEMS

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Abstract: The obstacle avoidance problem associated to a multi-steered general 1-trailer is studied in this work by considering the method of artificial potential fields. It is proposed a control strategy based on artificial potential fields that generate a trajectory to be followed by the tractor that represents, at the same time, a reference for the trailer. The local minima problem arising in presence of one or more obstacles is also analyzed.

Keywords: Mobile robots, obstacle avoidance, potentials.

1. INTRODUCTION

A trailer-like system is a mobile robot composed by a tractor which pulls n trailers connected by a revolute joint. There are two main models of trailer systems considered in the literature. Namely, the standard n-trailer and general n-trailer. The difference between these two models arises from the position of the joint. The joint of the standard n-trailer is located at the rear axle, while the joint of the general n-trailer is located off this axle.

The artificial potential field method was developed in (Khatib, 1986; Krogh et al., 1986), and has been extensively studied in the obstacle avoidance problem for autonomous mobile robots (Borestein et al., 1989; Ge et al., 2000; Tilove, 1990). There are two main approaches for this method. On one hand, the classical method (Khatib, 1986) depends only on the position of the mobile robot. On the other hand, the generalized potential field method (Krogh et al., 1986) depends also on the velocity of the robot. The underlying idea of the method is to fill the robot’s workspace with an artificial potential field in which the vehicle is attracted to its goal and is repulsed away from the obstacles.

One of the inherent problems of this method is the existence of local minima which are undesirable equilibrium points of a gradient system. They appear when the sum of the attractive and repulsive forces induced by the potential vanishes before its goal.

The multi-steered general n-trailer presented in (Orosco-Guerrero et al., 2002) is a trailer-like vehicle with actuated direction wheels for each trailer. In this work the obstacle avoidance problem associated to a multi-steered general 1-trailer is considered. This problem will be tackled by means of the method of artificial potential fields.

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2. KINEMATIC MODEL

A multi-steered general 1-trailer composed by a mobile robot and a trailer with a steered wheel is considered. The position of the center of the tractor axle with respect to the fixed reference frame \((X_1, X_2)\) is given by \((x_1, x_2)\). The orientation \(\theta_0\) corresponds to the angle that the robot forms with respect to \(X_1\), as shown in Figure 1. The orientation of the trailer is denoted by \(\theta_1\). The direction of the actuated wheels with respect to the longitudinal axis of the trailer is denoted \(\beta_1\). The variables \(u_1, u_2\) represent respectively the linear and angular velocities of the tractor and \(u_3\) represents the angular velocity of the steering wheel for the trailer. The position of the center of the tractor axle is given by \((w_1, w_2)\). The distances between the point \((x_1, x_2)\) and the joint is given by \(d_0\) and the distance between this joint and the point \((w_1, w_2)\) is denoted \(d_1\). With this notation the kinematic model is given by (Orosco-Guerrero et al., 2002),

\[
\begin{align*}
\dot{x}_1 &= u_1 \cos \theta_0 \\
\dot{x}_2 &= u_1 \sin \theta_0 \\
\dot{\theta}_0 &= u_2 \\
\dot{\beta}_1 &= a_1 (\theta_0, \theta_1, \beta) u_1 - a_2 (\theta_0, \theta_1, \beta) u_2 \\
\end{align*}
\]

where,

\[
\begin{align*}
a_1 &= \frac{1}{d_1 \cos \beta_1} \sin(\theta_0 - \theta_1 - \beta_1) \\
a_2 &= \frac{d_0}{d_1 \cos \beta_1} \cos(\theta_0 - \theta_1 - \beta_1).
\end{align*}
\]

3. ARTIFICIAL POTENTIAL FIELD METHOD

A gradient system (Hirsch et al., 1974) on an open set \(W \subset \mathbb{R}^n\) is a dynamic system of the form

\[
\dot{x} = -\nabla V(x)
\]

where \(V : \mathbb{R} \rightarrow \mathbb{R}\) is a \(C^2\) function, and

\[
\nabla V = \left( \frac{\partial V}{\partial x_1}, \ldots, \frac{\partial V}{\partial x_n} \right)
\]

is the gradient vector field, \(\nabla V : \mathbb{R} \rightarrow \mathbb{R}^n\) of \(V\).

Regular points of the trajectories of (2), cross level surfaces of the function \(V(x)\) orthogonally.

Denote the position of a point of the vehicle in a two-dimensional workspace by \(q = (y_1, y_2)\), the position of the goal by \(q_0 = (y_{1g}, y_{2g})\) and the position of a unique obstacle by \(q_o = (s_1, s_2)\). An artificial potential function applied to the vehicle at point \(q\), has the form,

\[
U(q) = U_a(q) + U_r(q),
\]

where \(U_a(q)\) is the attractive potential induced by the goal and \(U_r(q)\) is the repulsive potential induced by the obstacle. The resultant force is then obtained as,

\[
F = F_a + F_r,
\]

where,

\[
F_a(q) = -\nabla U_a(q), \\
F_r(q) = -\nabla U_r(q).
\]

In the sequel, \(\|\| : \mathbb{R}^2 \rightarrow \mathbb{R}^+\) denotes the usual Euclidean distance function.

The attractive force \(F_a\) guides the robot to the goal and \(F_r\) is a repulsive force which repels the vehicle from the obstacle. Usually the attractive potential is defined by,

\[
U_a(q) = \frac{1}{2} \xi \rho(q, q_0),
\]

where \(\xi\) is a positive scaling factor and \(\rho(q, q_0) = \|q - q_0\|^2\) is a positive definite function whose first derivative is continuous and has a global minimum equal to zero at \(q = q_0\).

The repulsive potential function takes the form,

\[
U_r(q) = \begin{cases} 
\frac{1}{2} \eta \left( \frac{1}{\rho(q, q_o)} - \frac{1}{\rho_0^2} \right)^2 & \text{if } \rho \leq \rho_0^2 \\
0 & \text{if } \rho > \rho_0^2 
\end{cases}
\]

where \(\eta\) is a positive scaling factor and \(\rho(q, q_o)\) is defined above. The region of influence of the obstacle is determined by the positive constant \(\rho_0^2\). Notice that \(U_r(\cdot)\) is once continuously differentiable, e.g. \(U_r(q) \in C^1(\mathbb{R}^2)\).

From equations (5, 6, 7), the induced forces can be expressed as,

\[
F_a(q) = -\xi (q - q_0) \\
F_r(q) = \eta \left( \frac{1}{\rho(q, q_o)} - \frac{1}{\rho_0^2} \right) \times \frac{\rho(q, q_o)^2}{\rho(q, q_o)} \frac{\partial \rho(q, q_o)}{\partial q}
\]

where \(F_r(q)\) is applied if \(\rho \leq \rho_0\) and is set to zero otherwise.
can be generalized by considering the force $F$ given by,

The coordinates of this point are that represents the middle of the front end of the tractor. The coordinates of this point are given by,

$$
\begin{align*}
\dot{y}_1 &= x_1 + \ell \cos \theta_0 \\
\dot{y}_2 &= x_2 + \ell \sin \theta_0,
\end{align*}
$$

where $\ell$ is the orthogonal distance between the rear axle point $(x_1, x_2)$ and the point $q$ of the robot. Taking the time-derivative of (9), produces,

$$\begin{bmatrix}
\dot{y}_1 \\
\dot{y}_2
\end{bmatrix} = A(\theta_0)
\begin{bmatrix}
u_1 \\
u_2
\end{bmatrix},\tag{10}
$$

with

$$A(\theta_0) = \begin{bmatrix}
\cos \theta_0 & -\ell \sin \theta_0 \\
\sin \theta_0 & \ell \cos \theta_0
\end{bmatrix}.
$$

Consider the desired values $\dot{q}_d = (\dot{y}_{1d}, \dot{y}_{2d})$ to be proportional to the normalized force generated by the potential field (Cadenat et al., 1999),

$$\begin{bmatrix}
\dot{y}_{1d} \\
\dot{y}_{2d}
\end{bmatrix} = \frac{v_d}{\sqrt{f_1^2 + f_2^2}}
\begin{bmatrix}
f_1 \\
f_2
\end{bmatrix},\tag{11}
$$

where $f_1, f_2$ are respectively the components of the total force $F$ in the direction of the axes $X_1$ and $X_2$ and $v_d$ is the desired scalar velocity. Under this conditions, by considering equations (10) and (11) it is possible to propose the feedback control law:

$$\begin{bmatrix}
u_1 \\
u_2
\end{bmatrix} = \frac{v_d}{\sqrt{f_1^2 + f_2^2}} A^{-1}(\theta_0)
\begin{bmatrix}
f_1 \\
f_2
\end{bmatrix}.\tag{12}
$$

Note that the attractive potential and the components of its negative gradient are given by,

$$U_a = \frac{1}{2} \xi \left((y_1 - y_{1g})^2 + (y_2 - y_{2g})^2\right)$$

$$f_{1a} = \xi (y_{1g} - y_1)$$

$$f_{2a} = \xi (y_{2g} - y_2).$$

On the other hand, the repulsive potential and the components of its negative gradient, inside of the region of influence, are given by,

$$U_r = \frac{\eta}{2} \left( \frac{1}{(y_1 - s_1)^2 + (y_2 - s_2)^2} - \frac{1}{\rho_0^2} \right)^2$$

$$f_{1r} = 2\eta \left( \frac{1}{(y_1 - s_1)^2 + (y_2 - s_2)^2} - \frac{1}{\rho_0} \right) \times$$

$$\frac{(y_1 - s_1)^2 + (y_2 - s_2)^2}{y_1 - s_1}$$

$$f_{2r} = 2\eta \left( \frac{1}{(y_1 - s_1)^2 + (y_2 - s_2)^2} - \frac{1}{\rho_0} \right) \times$$

$$\frac{(y_1 - s_1)^2 + (y_2 - s_2)^2}{y_2 - s_2}.$$

4. CONTROL STRATEGY

It is intended to take the vehicle from initial to a final position without colliding with obstacles. The control strategy will be developed in two steps. First, the tractor control law will be designed using an artificial potential field. Then the tracking problem of the trailer will be analyzed. By using the potential field method the tractor will follow the direction of the resultant force. The trajectory generated by the tractor is considered as a reference trajectory for the trailer.

4.1 Tractor Control

Consider the output function $q = (y_1, y_2)$ as the point that represents the middle of the front end of the tractor. The coordinates of this point are given by,

$$\begin{align*}
y_1 &= x_1 + \ell \cos \theta_0 \\
y_2 &= x_2 + \ell \sin \theta_0,
\end{align*}\tag{9}$$

Figure 2 shows the potential function for a three obstacles case. It is easy to see that the goal is a global minimum and the potential function indefinitely grows inside the region of influence of the obstacles.

Remark 1. Note that the classical method depends only on the relative position of the obstacles and vehicle. Notice that also the vehicle moves in the force field direction.

Figure 2. Artificial Potential Field Method.

4.2 Trailer Control

As mentioned before, it is intended that the trailer of the vehicle follows the path described by the tractor as a result of the artificial potential field applied to it. One way to achieve the above goal is to apply a control strategy that makes the center of the trailer axle $(w_1, w_2)$ to follow approximately the position of a point in the tractor $(y_1, y_2)$, subject to a given time delay $\tau$. This is, a point in the trailer has to take the same position that was generated by the tractor. To implement this strategy, consider $\beta$ as the output of the trailer subsystem. The angle $\beta$ will be manipulated to
Consider the error $e_\phi$ defined as,

$$e_\phi = (\phi - \phi_d),$$

where $\phi = \arctan\left(\frac{w_2-y_2(0)}{w_1-y_1(0)}\right)$ is the angle of point $(w_1, w_2)$ with respect to the initial position of the tractor and $\phi_d$ corresponds to its desired value. Note that the sign of the variable $e_\phi$ indicates in which side of the desired trajectory the trailer is placed.

In order to take into consideration the path generated by the potential field over the tractor (point $(y_1, y_2)$), the desired value $\phi_d$ is proposed as,

$$\phi_d = \arctan\left(\frac{y_2(t-\tau) - y_2(0)}{y_1(t-\tau) - y_1(0)}\right),$$

where the time delay $\tau$ is given by,

$$\tau = d_0 + d_1 + \ell v_d. \quad (13)$$

**Remark 2.** Note that due to the normalization of the forces generated by the potential field (11), the scalar velocity of the point $(y_1, y_2)$ is constant.

Now, it is possible to define a desired value for the output $\beta$, of the trailer subsystem, in terms of the relative position of the tractor and trailer by,

$$\beta_d = -ke_\phi,$$

from what, a control feedback law for the trailer is given by,

$$u_3 = -\alpha \tanh(\gamma(\beta - \beta_d)). \quad (14)$$

where $\alpha, \gamma$ are positive constants.

Figure 3 shows a block diagram of the complete control scheme (Tractor-trailer).

5. LOCAL MINIMA ANALYSIS

As mentioned before, an inherent limitation of the artificial potential field method is the existence of local minima. This is, points where the vehicle can stop. In this section the analysis of these equilibria will be developed.

It is clear that the method of artificial potential field can be analyzed as a gradient system.

The nonregular or critical points of $V$ are precisely the equilibria of system (2). Then nonregular points could be stable or unstable. Isolated minima are asymptotically stable.

In the case without obstacles, only the attractive force is present producing a global minimum at $q = q_g$. Since the case of one obstacle can be analyzed as a particular case of the one considering two obstacles, this second case will be analyzed in what follows.

A workspace containing two obstacles is shown in Figure 4. From this figure it is possible to see that two possibilities can arise. The one in which the regions of influence of the obstacles intersect each other and the one in which this phenomenon does not occur.

The non intersecting situation can be reduced to the one obstacle case (considered as a particular case). Therefore, for the two obstacle analysis, it will be assumed that the regions of influence are intersected. From Figure 4, by applying a change of coordinates (rotation and translation of the reference frame), the analysis of local minima for the configuration given by the reference frame $(X_1, X_2)$ always can be performed equivalently by considering the new reference frame $(X'_1, X'_2)$ with origin located at the middle of the obstacles.

**Remark 3.** Note that the equilibria must be located in the left half-plane, because in the right half-plane the attractive and repulsive forces are added.

The next theorem describes the situation where the goal is placed along the $X'_1$ axis (Figure 4).
Theorem 4. Consider the obstacle-goal distribution given in Figure 4 with the goal on the axis $X_1'$. Set $h = \frac{2\pi}{T}$ and $D = (-\rho_0, \rho_0)$. There exists a constant $h_0 \in \mathbb{R}^+$ such that system (1) satisfies one of the following statements:

a) For all $h \in [0, h_0)$, there does not exist any equilibrium points on the interval $D$ along $X_1'$.

b) If $h = h_0$, there exists exactly one (double) equilibrium point on the interval $D$ along $X_1'$.

c) For all $h \in [h_0, \infty)$, there exist two equilibrium points on interval $D$ along $X_1'$.

Sketch of Proof. The complete proof of the Theorem is a bit lengthy. Therefore, for the sake of conciseness, a simple geometric argument is presented. For the complete proof, the reader is referred to (Vidal-Calleja, 2002).

Considering the reference frame $(X_1', X_2')$, it is possible to write,

$$q = (x, 0), \quad q_g = (x_g, 0),$$
$$q_{o1} = (0, y_{o1}), \quad q_{o2} = (0, y_{o2}).$$

From Figure 4, it is clear that $y_{o1} = -y_{o2}$. Therefore, the force along the longitudinal axis $X_1'$ on the interval $D$ is given by,

$$f_1(x) = \frac{2h}{(x^2 + y_{o1}^2)^2} \left( \frac{1}{x^2 + y_{o1}^2} - \frac{1}{\rho_0^2} \right) x - (x - x_g).$$

The equilibria of the system occur precisely when $f_1(x) = 0$. After some algebraic manipulations, it is possible to show that the total force vanishes exactly at the zeros of the following function:

$$\gamma_1(x) = \kappa(x) + \lambda(x),$$

where $\kappa(x) = 2h \frac{\rho_0^2 - x^2 - y_{o1}^2}{(x^2 + y_{o1}^2)^3} \frac{1}{\rho_0^2}$ and $\lambda(x) = - (x - x_g)$.

Therefore, the force will vanish at the intersection of the functions $\kappa(x)$ and $-\lambda(x)$.

Figure 5 shows the typical shape of both functions. It is clear that, varying $h$, the function $\kappa(x)$ can be increased or shrunk so as to intersect $-\lambda(x)$ once or twice inside of the obstacles distance of influence. Note that zeros of $\kappa(x)$ are zeros of the repulsive force and are located at $x = \pm \sqrt{\rho_0^2 - y_{o1}^2}$ and $x = 0$. This shows that the equilibria lie within $D$. This completes the proof. \( \square \)

Example: Figure 6 is a bifurcation diagram of the equilibria, taking the following numerical parameters

$$x_g = 15, y_{o1} = 0.5, y_{o2} = -0.5, h = (0, 10], \rho_0 = 1.$$  

The case shown in Figure 4 where the goal is not on the $X_1$ axis can be treated in the same way of the Theorem 4. For this case Figure 7 shows its gradient potential field where a local minima can be observed.

6. SIMULATIONS RESULTS

To evaluate the obstacle avoidance control strategy developed before and in order to show that the methodology can be extended to the case of multiple obstacles, in this section a workspace with three obstacles will be considered.
The obstacles are located at $q_o(1) = (5, 5), q_o(2) = (7, 5), q_o(3) = (8, 8)$ and the goal was set at $q_g = (9, 10)$. The initial conditions of the vehicle was taken as $x_1(0) = 3, x_2(0) = 1, \theta_0(0) = 2, \theta_1(0) = 3, \beta(0) = 0$.

The simulation experiments were performed taking into consideration the real parameters of an experimental prototype with, $d_0 = 0.48, d_1 = 0.38, \ell = 0.3$.

The parameters considered in the feedback control law (12), (14) used for simulations are:

$$
\xi = 1, \eta = 2, \rho_0 = 1, \nu_d = 0.3, \alpha = 0.3, \gamma = 1, k = 50.
$$

The trajectories followed by the tractor and the trailer are shown in Figure 8, where it is possible to see how the path of the tractor represent, as expected, a reference for a trailer. In Figure 9, the control $u_1, u_2, u_3$, applied to the vehicle are depicted.

7. CONCLUSIONS

The obstacle avoidance problem associated to a trailer-like vehicle has been analyzed in this work. The study is done by considering a multi-steered general 1-trailer, a general model that present a steered wheel for the trailer. It is proposed a control strategy based on artificial potential fields that generate a trajectory that is followed by the tractor. This trajectory is considered as a reference for the control applied to the trailer. The performance of the control strategy was tested by simulation experiments. The local minima problem induced by the artificial potential field is also analyzed.

Fig. 7. Trajectory for the tractor-like vehicle

8. REFERENCES


