

Optimal Operation of Combined Heat and Power Systems: An Optimization-Based Control Strategy

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Abstract

The use of decentralized Combined Heat and Power (CHP) plants is increasing since the high levels of efficiency they can achieve. Thus, to determine the optimal operation of these systems in dynamic energy-market scenarios, operational constraints and the time-varying price profiles for both electricity and the required resources should be taken into account. In order to maximize the profit during the operation of the CHP plant, this paper proposes an optimization-based controller designed according to the Economic Model Predictive Control (EMPC) approach, which uses a non-constant time step along the prediction horizon to get a shorter step size at the beginning of that horizon while a lower resolution for the far instants. Besides, a softening of related constraints to meet the market requirements related to the sale of electric power to the grid point is proposed. Simulation results show that the computational burden to solve optimization problems in real time is reduced while minimizing operational costs and satisfying the market constraints. The proposed controller is developed based on a real CHP plant installed at the ETA research factory in Darmstadt, Germany.

Keywords: Combined heat and power systems, Profit maximization, Economic model predictive control, Mixed Integer Linear Programming, Non-constant time-step size

1. Introduction

The energy supply in Germany is increasingly changing from a centralized to a decentralized generation structure [1]. Besides the installation of renewable energies, the use of decentralized Combined Heat and Power (CHP) plants is increasing, as they can achieve high levels of fuel utilization [2]. However, the cost efficiency of a CHP is highly dependent on the operation strategy and, therefore, identifying an optimal operation strategy in volatile energy markets while covering thermal demands can be challenging though. To this end, optimization algorithms are often used for scheduling the activation instants of the CHP system, which can be determined according to the operating constraints of both the plant and the system itself [3].

In addition to operating constraints, the optimal operation of a CHP plant from an economic point of view is limited by the costs of the required resources (for instance, natural gas), the sale price of the electric power in the market, and its fluctuations over time. However, to take price fluctuations on the electricity market into account when choosing an operation strategy for CHPs, a sufficiently large optimization horizon is required. Particularly, for an intraday optimization, this horizon is usually assumed to be about one day, which is usually modeled by using a temporal resolution of 15 minutes that implies a high computational cost [4]. In [5], an optimization procedure is

proposed based on non-linear optimization techniques and it is applied to the determination of day ahead of operation program with a 15-minutes time step. On the other hand, in order to maximize the revenue by selling the generated electric power or its excess, it must be ensured that the plant actually produces the amount of electricity traded with the electric company to avoid economic penalties. From this fact, a higher temporal resolution to increase the degrees of freedom during the CHP operation, and more accurate modeling to achieve a proper representation of CHP behavior could be required [6]. Therefore, if both a long time horizon and a high temporal resolution are chosen, the number of decision variables taken into account in the optimization problem increases rapidly. This last issue is aggravated by an increased requirement for model accuracy when mapping the transient behavior of the plant. Thus, in order to solve the optimization problem within a short time, the number of considered decision variables must be kept low but in a way that allows the required accuracy.

According to the reported literature [7], the strategies proposed so far for planning and scheduling of CHP systems are focused on achieving a low operation cost, minimization of resources, or minimization of waste. However, most of these strategies are designed considering energy prices and the thermal and electric power demand to be constant for a long period of time. Thus, as a consequence, the CHP system is not able to be optimized in real-time since the current conditions of both the system and energy market will not be updated. In [8], one way to reduce the complexity of the problem is proposed by

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Nomenclature

Physics Constants

CHP	Combined Heat and Power
CVA	Canonical Variate Analysis
EMPC	Economic Model Predictive Control
EMPC ₁	EMPC controller with non-constant time step
EMPC ₂	EMPC controller with constant time step
LTI	Linear time-invariant
MOESP	Multivariable Output Error State Space
MPC	Model Predictive Control
N4SID	Numerical algorithm For Subspace Identification
RBC	Rule-based control
SI	Subspace identification
TES	Thermal energy storage

Number Sets

\mathbb{R}	Real Numbers
\mathbb{Z}	Integer Numbers
$\mathbb{Z}_{\geq 0}$	Positive integer Numbers

Symbols

Δt	Sampling time
\hat{x}	State estimation
$\mathbb{Z}_{\geq 0}$	Positive integer Numbers
\mathbf{u}	Input vector
\mathbf{v}	Measurement noise
\mathbf{w}	State noise
\mathbf{x}	State vector
\mathbf{y}	Output vector
\overline{P}_E	Upper bound for electric power generation
\overline{Q}_{TES}	Maximum thermal power in the TES
\overline{T}_2	Upper bound for output temperature in the CHP
θ_1	Cost by gas consumption
θ_2	Depreciation cost
θ_3	Switching cost
θ_4	Total revenues
θ_5	Difference between traded and produced electric power
\underline{P}_E	Lower bound for electric power generation
\underline{Q}_{TES}	Minimum thermal power in the TES
\underline{T}_2	Lower bound for output temperature in the CHP
$ \mathbf{x} $	Cardinality of vector \mathbf{x}

A_{TES}	Transfer area of TES
b	Number of input signals
C_p	Heat capacity
F_G	Volumetric flow of gas
F_W	Volumetric flow of water
f_{sw}	Switching indicator for the CHP system
H_p	Prediction horizon
J	Total profit
k	Discrete time index
l_e	Economic cost function
N	Model order
O_1	Extended observability matrix
p	Number of output signals
P_E	Electric power
$P_{r,e}$	Sale price of electric power
$P_{r,g}$	Gas price
$P_{r,on}$	Price of keeping the system on
$P_{r,s}$	Switching price
P_{tra}	Economic penalty in the traded power
P_{tra}	Traded electric power
q_{CHP}	Thermal power generated by CHP
q_{dem}	Thermal power demand
q_{loss}	Energy losses with the environment
Q_{TES}	Thermal energy stored in TES
T	Operation time
t	Continuous time
T_1	Temperature of water flow at the CHP inlet
T_2	Temperature of water flow at the CHP output
t_c	Computational time
T_s	Total simulation time
t_s	Controller time step
T_{env}	Outside temperature
T_{TES}	Temperature in the TES
T_{ud}	Updating period for traded electric power
U	Global coefficient of heat transfer
u	System input
x	Model state
\mathbf{V}^*	Optimal input sequence to CHP system
\mathbf{V}	Input sequence to CHP system

dividing it into a planning problem and a fulfillment or adaptation (tracking) problem. The longer-term energy marketing is considered in the planning problem and then given as a target to the tracking problem. In addition, a two-stage stochastic programming model for CCHP-microgrid operation considering demand uncertainty is proposed in [9]. However, with the introduction of the continuous intraday market, the boundaries between the planning and tracking phases might become blurred. Therefore, to make the best possible use of the resulting optimization opportunities on the market, an approach that combines the optimization of both problems in a single model

is required.

In this regard, optimization-based control strategies have gained attention since they allow considering the energy-price fluctuations and the system dynamics as constraints into an optimization problem behind the controller design. Among these strategies, the Model Predictive Control (MPC) approach has had a great application in the tracking stages once the planning stages have been previously optimized [10]. In [11], a stochastic MPC framework to optimally schedule and control the CHP microgrid with large-scale renewable energy sources to reduce the negative impacts introduced by uncertainties is proposed.

Nevertheless, due to the need to combine both planning and tracking problems as well as the increasing interest of improving the economic performance of the CHP systems, the Economic Model Predictive Control (EMPC) has gained attention during the last years with great applications in systems such as the boiler-turbine system[12], residential building energy systems [13], and mechanical pulping processes [14]. The main advantage of EMPC with respect to MPC is that the former directly optimizes an economic cost function of the process, from which both market constraints and time-varying price profiles could be considered into both the cost function and the constraints of an optimization problem [15, 16]. Thus, using the EMPC approach, both the planning and fulfillment problems could be addressed in one stage. Besides, according to the receding horizon principle [17] for the real-time implementation of control strategies, every time that the optimization problem is solved, the current cost/price information could be updated.

On the other hand, regarding model accuracy several approaches have been developed in the literature for the CHP plants, which are mainly focused on phenomenological-based models [18]. For instance, in [19], a modeling approach for a CHP plant that considers the effect of power gradients for depicts dynamic power changes more accurately than existing approaches is presented, while in [20], a multi-objective optimization model based on the technical, economic and environmental performances is developed. However, it is well known that phenomenological-based models require a high computational load because of the complexity of mathematical expressions used for representing the main phenomena that govern the system behavior. In addition, in most of the cases, model parameters, constants, or variables cannot be measured, estimated or determined in real environments. Consequently, data-driven models have become a useful alternative to model complex and large-scale systems [21]. Among the methods for getting models based on real data sets, the Subspace Identification (SI) algorithms were increasingly applied for the design of control strategies during the last years. The main reason for their widespread use is due to the fact that these methods directly deliver a linear state-space realization, which is quite useful in the design of control strategies [22]. In Table 1 some of the most relevant works during the last years are presented and classified according to the topics addressed for determining the optimal operation of CHP systems. From this review, it is possible to see that there is not a control strategy that integrates all the relevant features in only one strategy, apart of having real-time implementation capabilities.

Therefore, in order to solve the issues with respect to long optimization horizons, temporal resolution, and model accuracy, the main contribution of this paper is the design of a predictive-like controller based on the EMPC approach and by using a non-constant time-step size along the prediction horizon. The time-step width along the prediction horizon increases for time steps that lie further in the future, since if only the most recent time step is actually executed, model accuracy can be maintained while the number of decision variables is reduced. In this regard, the general idea of the proposed controller is to predict both the thermal and electrical power production that maximize

Table 1: Main research topics of the control strategies implemented for CHP systems.

Reference	One-stage strategy	Real-time implementation	Dynamic model	Predictive behavior	time-varying price profile	Trading constraints
Luo et al. [8]		✓	✓	✓		
Marino et al. [9]			✓			
Zhang et al. [11]	✓	✓	✓	✓		
Aluisio et al. [5]				✓	✓	
Costa and Fichera [4]	✓				✓	
Zhang et al. [3]	✓		✓		✓	
Proposed strategy	✓	✓	✓	✓	✓	✓

the profit during the CHP operation considering both operating and energy market constraints. Thus, the proposed controller will be designed based on a model for the operation of a CHP plant obtained by using SI methods and real data sets. In addition, in order to get the maximum profit regarding the sale of the electric power generated, a soft constraint for minimizing the difference between the generated and traded electric power is proposed, taking advantage of the high temporal resolution for the near future reached by using a non-constant time-step size.

It should be noted that in [23], a predictive-like controller was proposed using a non-constant time step as a first approximation to determine the optimal operation of cogeneration plants. However, in [23], energy losses due to the heat exchange with the environment and suitable measurements to avoid the high switching frequency of system actuators (e.g., valves, pumps) were not considered into the controller design. Thus, in contrast to the previous work, the proposed controller in this paper considers the energy losses in the process model, and in addition to energy market constraints and its fluctuations, safety constraints to avoid damages in the system are proposed in the controller design. Besides, the proposed controller is compared with another controller using a constant time step and more decision variables along the prediction horizon, in order to check the suboptimality of the proposed approach.

The remainder of the paper is organized as follows. In Section 2, the tools employed for the design of the proposed control strategy are briefly described. Then, the problem of maximizing profit for the operation of CHP plants is introduced in Section 3. Next, in Section 4, the proposed approach and the general idea about the non-constant time step are both presented and discussed. Then, a detailed description of the case study is presented in Section 5. Afterward, the obtained simulation results for the proposed approach are reported and analyzed in Section 6. Finally, conclusions and future work are drawn in Section 7.

2. Preliminaries

2.1. Economic Model Predictive Control

Model predictive control (MPC) is an optimization-based control technique in which a cost function is optimized over a prediction horizon according to a dynamic process model and process constraints. According to [24], the main idea underlying MPC is to transform a control problem into an optimization one, in a way that at sampling time a sequence of future control values is computed. Usually, MPC is formulated using a quadratic objective function to penalize the deviations of the state and outputs of a system from their optimal steady-state values over a prediction horizon [15, 10].

Although the conventional MPC approach has had a great application, it does not allow a suitable representation of the economic performance of the processes. According to [16], there is an increasing number of problems for which dynamic economic performance is crucial and the hierarchical separation of economic analysis and control is either inefficient or inappropriate. Thus, in order to perform in a joint manner the process economic optimization and process control, a new MPC scheme has been proposed, in which the conventional tracking function is replaced by an economic cost function and, it is called Economic MPC (EMPC). In this regard, given that the EMPC directly optimizes the process economics, it has been widely used in the context of the manufacturing industry with the aim to determine the optimal operation of manufacturing systems from an economic viewpoint.

According to [15], the EMPC approach is characterized by the following optimization problem:

$$\min_{u \in S} \int_0^{H_p} l_e(x(t), u(t)) dt \quad (1a)$$

subject to

$$\dot{x}(t) = h(x(t), u(t), 0) \quad (1b)$$

$$x(0) = x(t_k) \quad (1c)$$

$$g(x(t), u(t)) \leq 0, \forall t \in [0, H_p] \quad (1d)$$

being u the input trajectory of the decision variables over the prediction horizon H_p , x the predicted state trajectory, $h(\cdot)$ the mathematical expressions for the nominal process model, $x(0)$ the initial conditions on the dynamic model, $g(\cdot)$ the process constraints, and $l_e(\cdot)$ is the process economic cost function that the EMPC optimizes through dynamic operation of the process.

It should be noted that the implementation strategy of the EMPC is the same as for the conventional MPC, i.e., in a receding horizon fashion. Several works related to the design of EMPC controllers, the theoretical background and stability analysis of EMPC have been proposed in the literature. A detailed explanation of the EMPC strategy can be found in [16, 25]. On the other hand, some relevant applications of EMPC in industrial environments are presented in [26] for the building climate control in a Smart Grid, [12] boiler-turbine systems, and [14] for the mechanical pulping processes.

2.2. Subspace identification

Subspace identification (SI) methods allow identifying the matrices of a state-space realization of linear time-invariant (LTI) systems based on input-output data. These algorithms are useful since state-space realizations are convenient for estimation, control and prediction tasks. Many SI methods are based on algorithms that use both the observability and controllability matrices to determine the model matrices from input-output data for a real system.

Basically, SI methods start from the idea that a set of measurements of b input signals ($b \geq 1$) and p output signals ($p \geq 1$) satisfy an N -order state-space (unknown) realization as follow:

$$\mathbf{x}(k+1) = A \mathbf{x}(k) + B \mathbf{u}(k) + \mathbf{w}(k), \quad (2a)$$

$$\mathbf{y}(k) = C \mathbf{x}(k) + D \mathbf{u}(k) + \mathbf{v}(k), \quad (2b)$$

where $k \in \mathbb{Z}_{\geq 0}$ corresponds to the discrete-time index, $\mathbf{x} \in \mathbb{R}^\ell$, $\mathbf{u} \in \mathbb{R}^b$, and $\mathbf{y} \in \mathbb{R}^p$ are the state, input and output vectors, while $\mathbf{w} \in \mathbb{R}^\ell$ and $\mathbf{v} \in \mathbb{R}^p$ are the state noise and output measurement noise, respectively.

Thus, in a deterministic case, in which \mathbf{w} and \mathbf{v} are neglected, the SI problem consists of [27]

(a) Estimate the system order N .

(b) Estimate the system matrices $A \in \mathbb{R}^{\ell \times \ell}$, $B \in \mathbb{R}^{\ell \times b}$, $C \in \mathbb{R}^{p \times \ell}$, and $D \in \mathbb{R}^{p \times b}$.

According to [27], two different families of SI algorithms have been identified and reported in the literature. The first family uses the state estimation \hat{x} to determine the model matrices, while the second family uses the extended observability matrix O_i to first determine estimates of matrices A and C , and then, to estimate matrices B and D . Some examples of the first family of algorithms are the Canonical Variate Analysis (CVA) and Numerical algorithm For Subspace IDentification (N4SID), while a representative example of the second family is the Multivariable Output Error State Space (MOESP) algorithm. The algorithms of the first family are based on the system theory, the unifying theorem, linear algebra, and they can be generalized in two main steps:

1. Determine N and a state sequence $\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n, \hat{x}_{d+1}$. To this end, the data block Hankel matrix and the singular value decomposition are employed.
2. Solve a least-squares problem to obtain the state-space matrices based on state estimation $\hat{\mathbf{x}}$, and the measurements \mathbf{u} and \mathbf{y} .

The last procedure is deeply explained in [28], while some of the previously mentioned SI algorithms, and the way they are implemented, are widely explained in [22, 27].

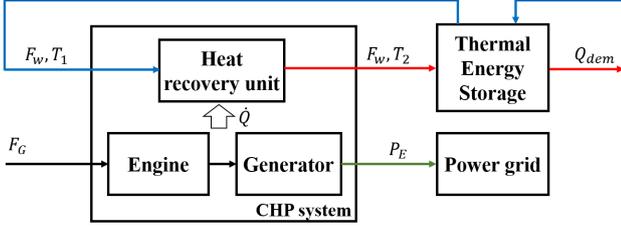


Figure 1: Combined Heat and Power (CHP) system with an integrated Thermal Energy Storage (TES) unit.

3. Problem Statement

The CHPs commonly refer to systems formed by a power generator and a heat recovery unit to produce electricity and useful heat at the same time. These systems are often based on gas engines to generate electric power and thermal energy for heating a water stream. In most cases, the heat recovered is used to directly cover the thermal power demand of buildings, other industrial processes, or it is transported towards a Thermal Energy Storage (TES) unit for its later use. A typical scheme of a CHP system with an integrated TES is shown in Figure 1.

According to Figure 1, the volumetric flow of gas F_G is fed to the CHP system for generating electric power P_E , which is usually used by the processes in the plant or injected in the local grid point of common connection for its sale. During the combustion process of F_G , the heat generated as waste is recovered by heating a cooling fluid (e.g. water) from temperature T_1 to T_2 via heat transfer. Afterward, the warm fluid is pumped towards the TES unit at a constant volumetric flow rate of F_W , from which the thermal power demand q_{dem} is covered. Thus, the thermal power recovered during the CHP operation q_{CHP} is stored in the TES, from which a water stream at the temperature of the TES T_{TES} is transported towards the industrial processes that require it.

Thus, in order to maximize the profit during the operation of a CHP system and to guarantee its proper operation, the costs associated to the resources consumption (e.g., F_G and F_W) should be minimized while complying q_{dem} and, the operating constraints of both the CHP system and TES unit are satisfied. In addition, the costs associated with the systems wear caused by high switching frequency and long operating hours of the CHP system could also be minimized in order to take into account the depreciation of the system.

On the other hand, besides to minimize costs related to the operation of a CHP system, incomes by the sale of the electric power generated could be generated. On the other hand, besides to minimize costs related to the operation of a CHP system, the incomes generated by the sale of the electric power could be maximized as a way to reduce the cost and take advantage of the system outputs. In this regard, in order to improve the profitability of the CHP system, the maximization of revenue could be performed taking into account the energy-price profile and its fluctuations in the market. Thereby, in order to determine the economic-optimal operation of a CHP system along an operation time T , the following control objectives are proposed:

3.1. Costs associated with the resource consumption

In this case, only the cost associated with the gas consumption is considered since the water flow in industrial environments is generally recirculated through the system as shown in Figure 1. Thus, the cost by gas consumption F_G is defined as follows:

$$\theta_1(k) = \sum_{k=1}^T F_G(k) P_{r,g}(k), \quad (3)$$

being $F_G \in \mathbb{R}_{\geq 0}$ the volumetric flow of gas consumed by the CHP system, and $P_{r,g} \in \mathbb{R}_{\geq 0}$ the gas price per day in suitable units. It should be noted that, for the case in which the water flow is not recirculated, an equation in the same way of (3) could be defined in terms of F_W .

3.2. Costs related to the CHP operation

Regarding the operation of the CHP system, two types of costs are considered:

- Costs of system depreciation due to the long operation hours of the CHP system, i.e.,

$$\theta_2(k) = \sum_{k=1}^T u(k) P_{r,on}, \quad (4)$$

being $u \in \{0, 1\}$ the current state (on/off) of the CHP system, and $P_{r,on}$ the cost of keeping the system turned on.

- Costs associated with the switching frequency of the CHP system f_{sw} , i.e.,

$$\theta_3(k) = \sum_{k=1}^T f_{sw}(k) P_{r,s}, \quad (5)$$

being

$$f_{sw}(k) = |u(k) - u(k-1)| \in \{0, 1\},$$

and $P_{r,s}$ a constant for the cost of switching on/off the system. It should be noted that θ_3 is also a way to penalize the high switching frequency of the CHP system to avoid damage system and respect its inertia.

3.3. Revenues for the sale of the electric power

For the cases in which the electric power generated P_E will be injected to the grid point common for its sale, the total revenues are defined as follows:

$$\theta_4(k) = \sum_{k=1}^T P_E(k) P_{r,e}(k), \quad (6)$$

being $P_E \in \mathbb{R}$ and $P_{r,e} \in \mathbb{R}_{\geq 0}$ the electric power generated and the sale price of P_E , respectively.

3.4. Trading of electric power

For the sale of P_E to any electric company, some market constraints must be satisfied to avoid economic penalties. In this regard, the amount of electric power traded with the electric companies for a fixed time period must be satisfied with small variations for the whole period. Thus, in order to satisfy the traded electric power, a new control objective that considers the energy market constraints must be included. Assuming that the electric power traded with the electric company P_{tra} has an updating period T_{ud} , for which P_{tra} must be approximately equal to P_E , the control objective proposed to minimize differences between P_E and P_{tra} along T_{ud} is defined as follows:

$$\theta_5(k) = P_{r,\Delta P} \left(P_{tra}(k : T_{ud}) - \sum_{i=k}^{i=T_{ud}} P_E(i) \right), \quad (7)$$

being $P_{tra}(k : T_{ud})$ the traded power from the time instant k up to the next updating period T_{ud} , $i \in \mathbb{Z}_{\geq 0}$, $P_{tra} \in \mathbb{R}_{\geq 0}$, $T_{ud} < T$, and $P_{r,\Delta P}$ the economic penalty for not fulfilling P_{tra} .

It should be noted that, according to (3) - (7), $F_G(k)$, $F_W(k)$ and $u(k)$ refer to the inputs of the CHP system, while $P_E(k)$ is one of the outputs resulting from feeding such inputs to the system. Thus, according to the previous discussion, in order to determine the optimal operation of a CHP system with an integrated TES (see Figure 1), the economic cost function proposed in this work is defined as follows:

$$J(k) = -(\theta_4(k) - \theta_1(k) - \theta_2(k) - \theta_3(k) - \theta_5(k)), \quad (8)$$

being $J \in \mathbb{R}$ the total profit for the CHP operation along T , which corresponds to the revenues for the sale of P_E (θ_4) minus the total costs ($\theta_1, \theta_2, \theta_3, \theta_5$) to operate the CHP system.

Thus, in order to achieve the control objective in (8), the activation/deactivation instant of the CHP system and the optimal amount of gas to feed the system should be determined. However, to compute both the revenue and total costs, suitable dynamic expressions are required for both relating the input (F_W, T_1, F_G, u) and output (P_E, T_2) variables of the CHP system, as well as for modeling the heat transfer in the TES unit, i.e.,

$$\begin{aligned} \psi(k+1) &= f_1(\psi(k), F_W(k), T_1(k), F_G(k), u(k)), \\ P_E(k) &= f_2(\psi(k)), \\ T_2(k) &= f_3(\psi(k)), \end{aligned} \quad (9)$$

being $\psi \in \mathbb{R}$ the state vector in the corresponding linear/non-linear map $f_1 : \{0, 1\} \times \mathbb{R} \mapsto \mathbb{R}$, while $f_2, f_3 : \mathbb{R} \mapsto \mathbb{R}$ are the linear/non-linear maps that relate the states ψ with the output variables for the CHP system.

In addition to the dynamics for the operation of the CHP system, the dynamic for the thermal energy stored in the TES should be modeled in order to guarantee the thermal power demand q_{dem} and satisfy operating constraints. In this regard, a simplified model based on the energy conservation principle is proposed considering the energy losses by heat exchange with the environment. According to the scheme in Figure 1, at each time that the CHP system is activated, a water stream with flow

F_W is pumped from the TES towards the heat recovery unit in the CHP system for its heating. Afterward, this warm flow is returned to the TES where a phenomenon of heat transfer takes place increasing or decreasing the temperature inside of TES T_{TES} according to the amount of water stored and the others input-outputs flows. One of the output flows of the TES unit is the stream pumped to cover the required q_{dem} . It should be noted that, for this case, the temperature inside the tank is assumed to be uniform, it means the temperature of the output streams will be equal to the temperature inside of TES, i.e., $T_1 = T_{TES}$. It worth noting that each flow of hot water pumped from the TES, e.g., towards the CHP system and the building for covering q_{dem} , is returned to the tank after to comply with their functions.

Thus, in terms of heat flow, the dynamic for the energy stored in the TES unit could be defined as

$$Q_{TES}(k+1) = Q_{TES}(k) + \Delta t (q_{CHP}(k) - q_{dem}(k) - q_{loss}(k)), \quad (10)$$

being

$$q_{CHP}(k) = F_W(k) C_p (T_2(k) - T_1(k)), \quad (11)$$

the heat flow per time unit recovered from the CHP system and,

$$q_{loss}(k) = U A_{TES} (T_1(k) - T_{env}(k)). \quad (12)$$

the heat flow per time unit lost by heat transfer with the environment at instant k .

In (10)-(12), $q_{dem} \in \mathbb{R}$ refers to heat flow per time unit required by other processes, $Q_{TES} \in \mathbb{R}$ is the heat stored in the TES unit, T_1 and F_W are measured inputs, C_p is the heat capacity of water, A_{TES} corresponds to the transfer area of the TES unit, U is the global coefficient of heat transfer, T_{env} is the surroundings temperature, and Δt refers to the sampling time. Besides, taking into account the physical constraints of the TES unit, it must satisfy $\underline{Q}_{TES} \leq Q_{TES} \leq \overline{Q}_{TES}$ for all k , being \underline{Q}_{TES} and \overline{Q}_{TES} the lower and upper bounds of Q_{TES} , respectively. Finally, in addition to the process models, other operating constraints such as operating ranges, among others, could be considered in order to guarantee the proper operation of the CHP system.

4. Proposed Approach

In order to determine the optimal operation of the CHP system from an economic point of view, the EMPC approach is addressed to design an optimization-based controller that allows maximizing the revenue while minimizing the operating costs along a prediction horizon. From the EMPC approach, the control problem can be transformed into an optimization problem, in which process dynamics, operating limitations, and control objectives will be considered like the constraints and cost function of an optimization problem, respectively. It should be noted that due to the predictive behavior of the EMPC approach, process dynamics are required to perform the prediction of system behavior along a prediction horizon. In this regard, the controller design, the philosophy behind the use of a non-constant

time step along the prediction horizon, and the model process for the CHP system will be explained below.

4.1. Controller design

Based on the receding horizon philosophy [17] and considering a fixed prediction horizon H_p , the general idea is to predict the input sequences of the system $v(k) = [u(k) F_G(k)]^T$ that maximize the profit taking into account both the operating constraints and the time-varying price profiles. Thus, according to the defined control objective in (8), sequences for J and system inputs v along H_p can be defined as follow:

$$\mathbf{J}(k) \triangleq \{J(k|k), \dots, J(k+H_p-1|k)\}, \quad (13a)$$

$$\mathbf{V}(k) \triangleq \{v(k|k), \dots, v(k+H_p-1|k)\}, \quad (13b)$$

$$(13c)$$

with $\mathbf{J} \in \mathbb{R}^{H_p}$ and $\mathbf{V} \in \{0, 1\}^{H_p} \times \mathbb{R}^{H_p}$. Thereby, the economic predictive-like controller is based on the following open-loop optimization problem:

$$\min_{\mathbf{V}(k)} \mathbf{J}(k) \quad (14a)$$

subject to (9) - (10) and

$$F_w(k|k) = F_w u(k|k), \quad (14b)$$

$$u(k|k) \in \{0, 1\}, \quad (14c)$$

$$Q_{TES}(k|k) \in [\underline{Q}_{TES}, \overline{Q}_{TES}], \quad (14d)$$

$$T_2(k|k) \in [\underline{T}_2, \overline{T}_2], \quad (14e)$$

the logical conditions:

$$u(k|k) = 1 \iff F_G(k|k) \in [\underline{F}_G, \overline{F}_G], \quad (14f)$$

$$u(k|k) = 1 \iff P_E(k|k) \in [\underline{P}_E, \overline{P}_E], \quad (14g)$$

$$u(k|k) = 0 \iff P_E(k|k) = 0, \quad (14h)$$

$$u(k|k) = 0 \iff F_G(k|k) = 0, \quad (14i)$$

and the following safety constraints to avoid the chattering effect and to smooth the behavior of system actuators:

$$\Delta u(k|k) = 0 \iff 0 \leq \Delta F_G(k|k) \leq \varepsilon, \quad (14j)$$

$$\Delta u(k|k) = 1 \iff 0 \leq \Delta F_G(k|k) \leq \mu, \quad (14k)$$

being $\Delta u(k|k) = |u(k|k) - u(k-1|k)|$, $\Delta F_G(k|k) = |F_G(k|k) - F_G(k-1|k)|$, F_w the constant flow of water to recover the heat produced by the CHP system, P_E the electric power generated, ε the maximum variation allowed in F_G when the system remains on, μ the maximum variation allowed in F_G when the system switches on, and $\underline{P}_E, \overline{P}_E, \underline{T}_2, \overline{T}_2, \underline{F}_G, \overline{F}_G$, and $\underline{Q}_{TES}, \overline{Q}_{TES}$ the lower and upper bounds for P_E, T_2, F_G and Q_{TES} , respectively. Thus, taking into account the nature of variables to be optimized, the optimization problem in (14) is a mixed-integer linear programming (MILP) problem, for which suitable solvers should be

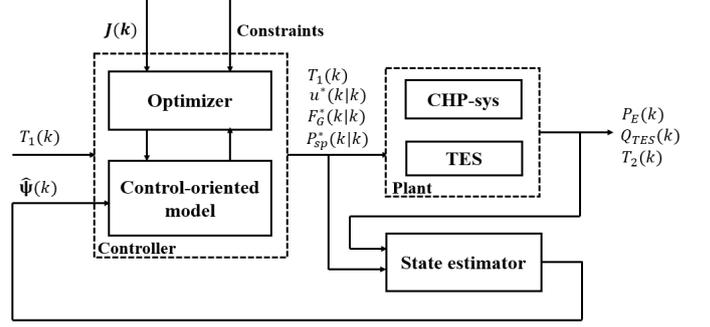


Figure 2: Control scheme for the optimal operation of CHP system with an integrated TES.

chosen in order to solve the problem with a low computational burden.

Assuming that the optimization problem defined in (14) is feasible, i.e., $\mathbf{V}(k) \neq \emptyset$, the optimal sequence $\mathbf{V}^*(k)$ exists and, according to receding horizon approach, the first component $v^*(k|k) = [u^*(k|k) F_G^*(k|k)]^T$ is sent to the plant. Then, this procedure is repeated for the next instant $k+1$ once measurements of input signals and estimation of the required information about the plant are updated for the next iteration. In order to obtain suitable estimations of model states for the next iterations, an state estimator could be required. The proposed control scheme in real time to determine the optimal inputs along H_p is presented in Figure 2.

According to Figure 2, the optimization problem in (14) is solved in real time into the controller module in order to determine the optimal sequence $\mathbf{V}^*(k)$. Next, the first components, i.e., $u^*(k|k)$, $F_G^*(k|k)$, are sent to both the plant and the state estimator. Then, from these signals as well as the real measurements from the CHP system, the current estimation of the model states $\hat{\Psi}(k)$ is performed by the estimator and fed back to the controller for the next iteration. Thereby, to estimate the states of the CHP system, a Kalman filter based on the real measurements of the outputs of the CHP system will be designed.

It should be noted that since the electricity trading is usually accounted for and fulfilled every 15 minutes, an unfulfillment of electrical power at the beginning of this time slot/window could be compensated at the end of such time window. Thus, at every time step, the system inputs can be adjusted in order to reduce the difference between the amount of energy traded P_{tra} and produced P_E . Taking into account that the way of this restriction is satisfied is one of the contributions of this paper, the trajectory of the prediction model should be updated at time slots lower than a quarter of an hour with the aim to have more degrees of freedom along T_{ud} . This last requirement is satisfied using a non-constant time-step size along H_p considering time slots lower than 15 minutes during the first part of H_p , and then increasing the step size to avoid increasing the computational load.

Thus, assume that at each time instant k an optimal sequence is found along H_p and the predicted P_E from k up to T_{ud} is sold to the electric company. Then, for next iteration $k+1$, the

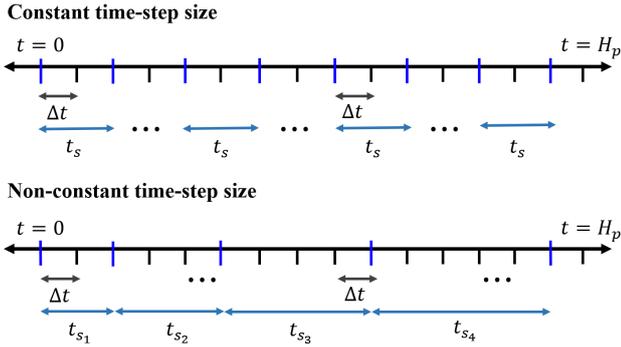


Figure 3: Scheme for the approaches of both constant and non-constant time-step sizes along H_p .

objective in (8) is minimized in order to guarantee that the sold electric power P_{tra} can be achieved at instant T_{ud} , considering the P_E produced from k up to $k+T_{ud}$. Next, when $k = T_{ud}$, a new amount of electric power to be sold is predicted and updated to the electric company, and the process is repeated once again. Thereby, θ_5 is considered only along the first 15 minutes of H_p since P_{tra} for this time period has already sold while for the rest of H_p new targets can be defined according to the variations of the energy market.

4.2. Non-constant time-step size

Due to the high computational cost for solving optimization problems with a large number of variables and the need to solve them fast enough for their implementation in real time, in this paper a non-constant time-step size along H_p is proposed. The general idea is to reduce the number of decision variables along H_p by using a shorter time-step size at the beginning of H_p and increasing the time-step size towards the end of H_p . That means a greater number of decisions will be considered in the near future while fewer decision variables are considered in the far future. It is worth noting that, the time-step size does not refer to the sampling time Δt , which is defined by sensing devices and used in the model identification. In contrast, the controller time step, called henceforth t_s , corresponds to the time instants along H_p in which the controller makes decisions, with $t_s \geq \Delta t$. A representation of the proposed approach is shown in Figure 3.

Thus, for a given Δt , which can be established according to the data acquisition devices in the plant or in the modeling procedures, the proposed approach considers using a non-constant t_s along H_p greater than Δt . Therefore, according to Figure 3, the controller can only make decisions at the time instants corresponding to the time steps $t_{s3} \gg t_{s2} \gg t_{s1} \geq \Delta t$ and not at each Δt . It should be noted that, in the strict case $t_s > \Delta t$, an internal loop is required, e.g., if $\Delta t = 0.1s$ and $t_s = 1s$, ten iterations for the process model should be implemented. This procedure is commonly known as blocking [17].

Finally, since the optimization problem in (14) explicitly considers models for the generation of both electric and thermal power (9), suitable expressions for characteristic maps f_1 , f_2 and f_3 in (9) are required. Taking into account the complexity

of these systems, this paper proposes the identification of data-driven models by using SI methods. The procedure followed to identify the model of the CHP system is explained in the next section.

4.3. Model of a Combined Heat and Power System

Although the expressions for the dynamics that take place in the TES unit are given in (10)-(12), suitable expressions for modeling the CHP system will be determined based on real data. In this paper, the SI methods have been selected for model identification as they allow to obtain a state-space representation from input-output data [29, 21, 22]. Thus, a linear approximation for the dynamics of the CHP system (i.e., maps f_1 , f_2 and f_3) is considered.

According to Section 2.2, in order to determine the model matrices and the order N , this paper focuses on the N4SID algorithm to get proper expressions for maps f_1 , f_2 and f_3 due to its great application and implementation in software [22, 28]. Therefore, to obtain a suitable state-space representation, proper experiments should be performed for getting information about the real system behavior at different operating conditions. Thus, according to the process shown in Figure 1, suitable tests were performed feeding different values of F_G and T_1 to the CHP system in order to get its dynamic response and its corresponding outputs. It should be noted that for this case, changes in the water flow were not considered since this flow is considered constant always the CHP system is turned on. However, for those cases in which F_W can be modulated, experiments changing this flow could also be required in order to represent the real behavior of the system. Besides, when F_W can take different values, it can be added as a decision variable into the optimization problem (14) considering the associated costs.

5. Case study

The system to be considered in this paper is based on a real CHP system with an integrated TES unit, which is in the ETA research factory at Technische Universität Darmstadt, Germany. As shown in Figure 1, although the CHP system is formed by different devices, in this paper, the model for each component of the CHP system is not addressed separately. Instead of that, all elements are considered as a joint system, i.e., the CHP. Following this idea, the CHP system will have three inputs, namely, T_1 , F_W and F_G , which feed the system to produce the outputs P_E and T_2 . Although the direct output of the CHP system is T_2 , from this variable, the heat flow recovered q_{CHP} can be computed by using (11).

The CHP system considered has a maximum electric power capacity $P_{E,max} = 6$ kW when the maximum flow of gas $F_{G,max} = 2.4$ m³/h is fed to the system. Thus, when the system is turned on, the produced P_E is sold to the power grid in order to maximize revenue and mitigate the cost associated with the thermal power production q_{CHP} , which is the main objective of the CHP system in the ETA research factory. It should be noted that a pump with a constant flow rate of $F_W = 2753.4$ L/h is

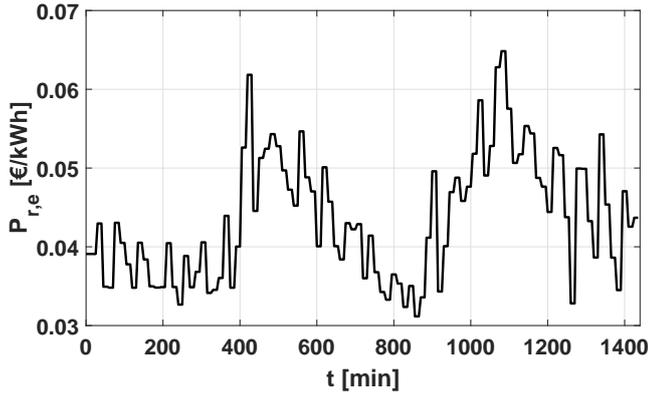


Figure 4: Daily energy price profile for the sale of P_E .

assumed, and since the fluid is recirculated through the system the associated costs are neglected. The maximum capacity of the TES unit in terms of energy is equal to $\bar{Q}_{TES} = 1000\text{kWh}$.

On the other hand, in order to get real data, the CHP system is equipped with several sensing devices that provide information in real time about its inputs and outputs. Due to the nature of these systems, their settling times, and the number of variables to be sensed, a sampling time equal $\Delta t = 10\text{s}$ was chosen to provide data about the real system operation. This information will be useful for both model identification and its validation.

In addition, a prediction horizon of $H_p = 24\text{hours}$ was established to test the proposed approach for one operation day. It means a simulation time of $T_s = 24\text{hours}$. Thus, taking into account the energy market fluctuations, the daily energy sale price profile presented in Figure 4 was considered into the optimization problem to compute the revenue achieved for the sale of P_E .

6. Simulation results

6.1. Model identification

According to Section 4.3, the process model for the analyzed CHP system was identified by using the routine `n4sid` of the System Identification ToolboxTM provided by Matlab[®]. Thus, according to real-data sets from the CHP system in the ETA factory, different values of model order N were tested, and then, the model matrices A , B , C , and D were identified with the aim to achieve a high fit degree between the measured and modeled outputs. The model validation for the system outputs is presented in Figure 5.

From the results in Figure 5, it is possible to see that the obtained model properly represents the behavior of both thermal and electric power productions in a suitable way. According to the state-space representation in (2), the identified model matrices are presented in the Appendix.

6.2. Proposed control scheme

Based on the case study presented in Section 5 and the proposed control approach in Section 4, a non-constant time-step

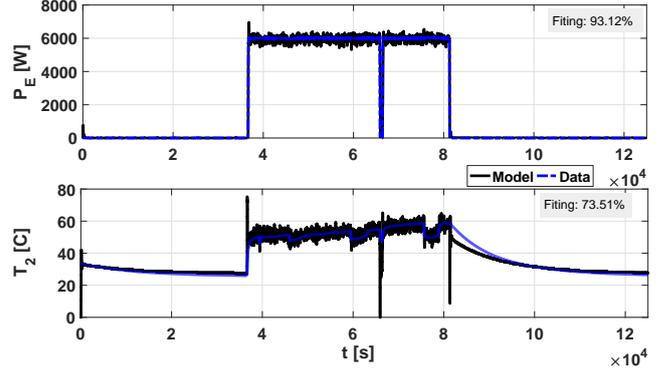


Figure 5: Validation of the CHP model obtained by using SI methods.

size t_s along H_p is proposed for reducing the computational burden. Thus, given that $\Delta t = 10\text{s}$ and considering H_p as one day, with a traditional predictive control approach a total of 1440 decision variables along H_p should be computed for each input variable (u and F_G). However, the proposed approach states the following decision distribution along H_p :

- From $t = 0$ to $t = 30$ minutes, $t_s = 5$ minutes and then six elements of the sequences $\mathbf{V}(k)$ are computed.
- Then, from $t = 30$ to $t = 60$ minutes, $t_s = 15$ minutes is fixed and two elements of $\mathbf{V}(k)$ are computed.
- After one hour, $t_s = 1$ hour and the rest 24 elements of $\mathbf{V}(k)$ are computed.

Finally, $|\mathbf{V}(k)| = 32$, which corresponds to $H_p = 24$ hours. On the other hand, since the model for the CHP system runs each ten seconds based on the real data, as explained in Section 4.2 a faster internal loop with an adaptive length that is updated according to the current value of t_s was included in the controller design. In this sense, the proposed controller will be executed every five minutes by using a prediction horizon of one day ahead and including the prediction of both the model dynamics and the operation cost according to the current energy market conditions. Then, according to the execution time of the controller, the optimal activation/deactivation sequence of CHP is computed every five minutes taking into account the update of information required by the controller such as the current state of the CHP system and energy market. It means that at every five minutes new optimal scheduling is computed for the next 24 hours.

Thus, given the mixed-integer linear programming nature of the optimization problem in (14), and the need to solve this problem fast enough to react in real time, simulations were developed in Matlab[®] using the solver IBM ILOG CPLEX Optimization Studio [30] and YALMIP toolbox [31] for stating the optimization problem in the specific format of the solver. Besides, simulations were performed using a processor Intel[®] CoreTM i7-5500U CPU 2.40GHz and RAM of 8.0 GB and the parameters in Table 2, in which the costs are presented in economic units (e.u.).

Table 2: Simulation parameters for the CHP system operation.

Parameter	Value	Parameter	Value
$P_{r,g}$	0.0218 e.u./kWh	P_{gas}	11.384 kW/m ³
$P_{r,on}$	5 e.u./kWh	$P_{r,s}$	5 e.u./kWh
\overline{Q}_{TES}	1 MW	\underline{Q}_{TES}	2 kW
C_p	4.180 kJ/kmol K	ρ	1000 kg/m ³
\overline{T}_2	25 °C	\underline{T}_2	80 °C
\overline{F}_G	0.5 m ³ /h	\underline{F}_G	2.4 m ³ /h
\overline{P}_E	3000 w	\underline{P}_E	6000 w
ε	0.3 m ³ /h	μ	2.4 m ³ /h

The proposed control approach (EMPC) was compared with a typical rule-based control (RBC) implemented in these systems. The underlying idea from RBC is that every time that the system searches one of the operating bounds, the system will be switching on/off depending on the bound achieved. It should be noted that since the RBC does not have predictive behavior, the control objective related to the trading of P_E (θ_5) was not considered when the total operating costs were computed. For the rest of the control objectives, the comparative was performed using the same operating constraints and measurements of the temperature of T_1 .

In Figure 6, the optimal sequences for the activation of the CHP system, and both the water and gas flow fed to the CHP system are presented. From these results, it is possible to observe that RBC has a higher switching frequency than EMPC, which increases the wear of systems and therefore the associated costs. Besides, for each activation of the CHP system, the optimal value found for F_G using the EMPC approach presents some small variations that allow modulating the q_{CHP} produced without decrease significantly the generation of P_E . This last fact could be a consequence of the trade-off between the control objectives of maximizing revenue for the sale of P_E and compensate for the total costs. Then, according to the optimal input sequences, in Figure 7, the corresponding outputs of the system are shown. Thus, based on these results, in order to maximize the sale of P_E , the optimal sequence of F_G was set near its upper bound with some small variations to avoid violating the operating range of the TES unit and satisfying q_{dem} .

In addition, from Figure 7 it should be noted that the produced q_{CHP} is always higher than q_{dem} when the CHP system is turned on. That means the excesses in the thermal power production are stored in the TES for their later use when the system is turned off or even for supplying an unexpected demand. However, for the case in which the EMPC is implemented, the modulation in the values of F_G allows that the upper bound of Q_{TES} will be achieved in greater time than when the RBC is implemented. From this fact, it is possible to keep the system turned on along a long time avoiding turning off the system when reaching its bounds.

On the other hand, in Figure 8, the temperature profiles of the measured T_1 inside the TES unit, and the obtained T_2 using both EMPC and RBC are presented. According to the results shown in Figure 8 and the obtained values of F_G for each ap-

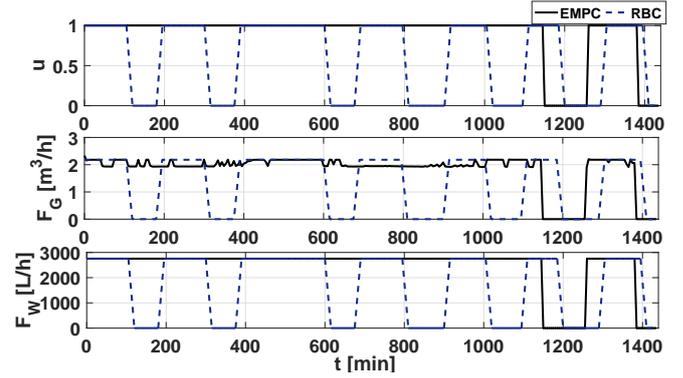


Figure 6: Optimal sequences for the CHP system activation (a), gas flow (b) and water flow (c) for both control strategies.

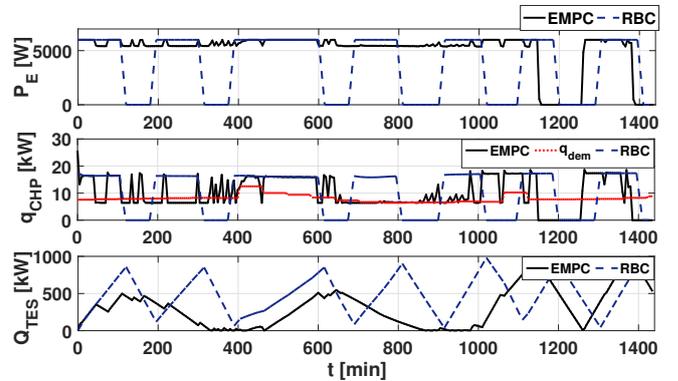


Figure 7: Generated electric power (a), production of thermal power (b), and thermal power level in TES (c) for both control strategies.

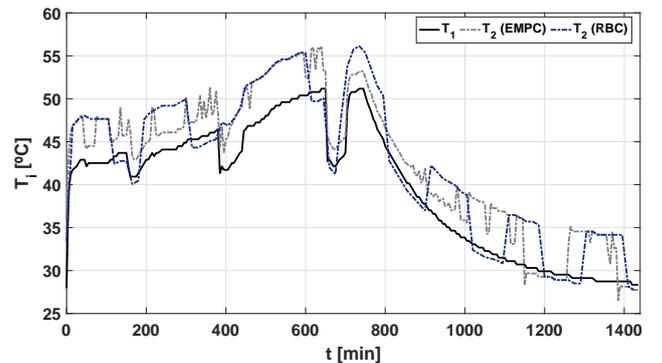


Figure 8: Temperature profiles of both the water inside the TES (T_1) and of F_W at the output of the CHP system (T_2).

proach, it is possible to see that higher values of T_2 are achieved using the RBC. This fact is given since for the RBC approach, the gas flow cannot be modulated and is always pumped at the maximum admissible value.

According to the control objective for the sale of P_E in the market and the energy trading constraints, Figure 9 shows a comparison between the generated P_E and the traded P_{tra} at each 15 minutes. From these results, it is possible to conclude

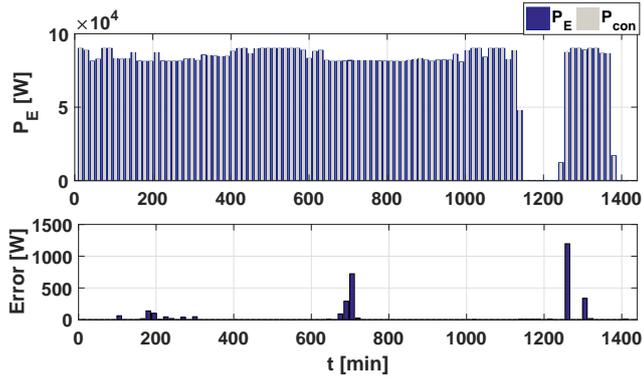


Figure 9: Comparison between P_E and P_{tra} using the proposed approach (a), and its corresponding error (b).

that a high fitting degree between P_E and P_{tra} can be achieved with an error percentage near 0.8% when the proposed control strategy is implemented. Thus, by using the proposed EMPC approach, it is possible to minimize the differences between P_E and P_{tra} due to the degree of freedom of the controller for adjusting the difference every five minutes before to reach the next T_{ud} . It should be noted that results are only shown for EMPC strategy because by using the RBC it is not possible to get a prediction of electric power generation, and therefore, economic penalties cannot be avoided. Thus, including θ_5 into the optimization problem, the economic penalties can be reduced or even avoided if a suitable analysis of deviations between these values is performed and reported to the electric company.

Afterward, taking into account the importance to solve the optimization problem fast enough for the implementation in real time of the proposed approach, in Figure 10 the computational time t_c spent by iteration along T_s is presented. It worth noting that for each iteration, a $H_p = 24$ hours is considered with the suitable update of price profiles and thermal power demand. Thus, from obtained results, it can be observed that even for a great number of decision variables and long prediction horizons, values of t_c lower than five seconds could be achieved when the proposed controller with a non-constant time-step size is implemented. Then, since the time spent on solving the optimization problem is always lower than ten seconds, and according to the controller design the optimal inputs are found and sent to the CHP system at every five minutes, the proposed control strategy is suitable for its implementation in real time. This last fact is a direct consequence of the use of the non-constant time-step size along the prediction horizon proposed in this paper.

Finally, in Table 3 the operating costs, revenue, and the real cost for one day of operation using both the proposed EMPC and RBC strategies are reported. From these results, it is possible to see that even if the economic penalties by the difference between P_E and P_{tra} are neglected when the RBC is used, cost reductions per day around 11.3% can be achieved when the proposed EMPC is implemented. Besides, it should be noted that more revenue can be achieved by selling the electric power generated when the proposed controller is implemented since the

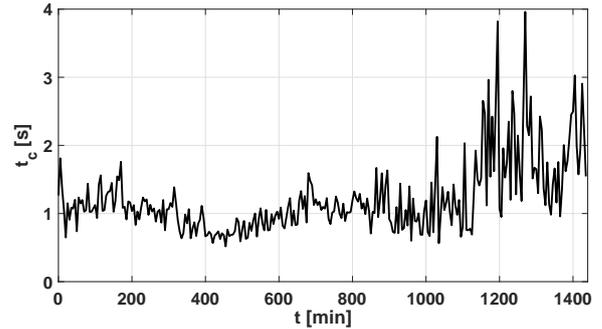


Figure 10: Computational time by iteration along T_s .

Table 3: Total costs in economic units for one operation day by using both control strategies.

Approach	Operating costs	Revenue	Real costs
RBC ¹	151.73 e.u.	3.86 e.u.	147.87 e.u.
EMPC	137.06 e.u.	5.23 e.u.	131.80 e.u.

fluctuations of energy prices are considered and updated every five minutes into the optimization problem. Thus, according to the price profile, the controller makes the decision to produce more or less P_E . However, it should be noted that the sale of electric power to the grid point is only a strategy to take advantage of one of the outputs of the CHP system and is not the main objective of this proposal. Therefore, the real costs for the operation of the CHP system in Table 3 were computed as the total operating costs minus the income achieved by the sale of P_E .

6.3. Comparative assessment

In this section, the proposed EMPC controller with non-constant t_s (henceforth associated to EMPC₁) is compared with another EMPC controller but considering a constant t_s (EMPC₂). The underlying ideas behind this comparative assessment are to evaluate the performance of the proposed controller when disturbances take place and to study the lost of optimality due to the reduction in the decision variables in EMPC₁ with respect to EMPC₂. In order to test both controller designs under the same operating conditions, a new scenario was designed considering disturbances during the CHP operation.

Thus, the general idea is to show the advantages of using a non-constant time-step size (EMPC₁) defined with shorter time-step sizes at the beginning of H_p , and, based on the obtained results, to compare the performance of the proposed controller with respect to the conventional EMPC₂. This last fact is given since the total number of decision variables along H_p is reduced even when more degrees of freedom are considered for the near instants when EMPC₁ is used. In this regard, the EMPC₂ was designed using a constant time step $t_{s,2} = 30$ minutes along the whole H_p , i.e., $|V(k)| = 48$. It should be noted that $t_{s,2}$ has been

¹It should be noted that in this case, economic penalties by not satisfying the trading of electric power are not included.

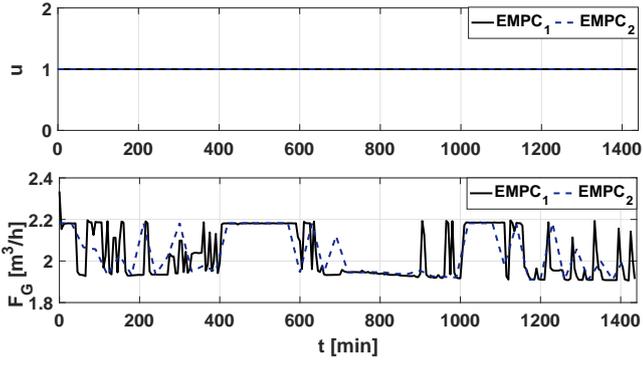


Figure 11: Optimal sequences for the inputs of the CHP system by using EMPC₁ and EMPC₂.

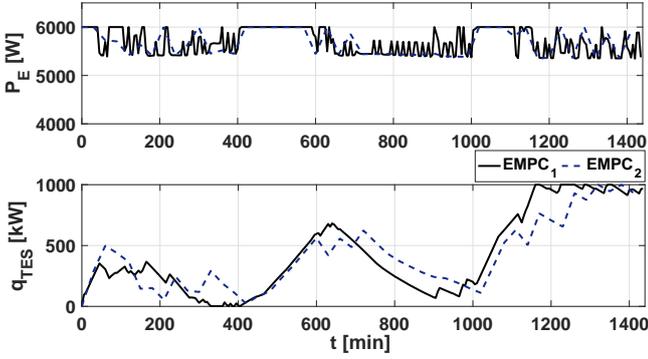


Figure 12: Optimal sequences for the outputs of the CHP system by using EMPC₁ and EMPC₂.

selected larger than $t_{s,1}$ since for lower values of $t_{s,2}$ it was not possible to finish the simulation due to the high computational load.

Two disturbances to simulate a fault in the valve that provides the gas flow (F_G) to the CHP system were simulated. Thus, from $t = 60$ minutes up to $t = 120$ minutes, a reduction of 5% in the optimal value of F_G fed to the CHP system was considered. Afterwards, in the same way as the first disturbance, from $t = 1000$ to $t = 1120$ minutes a reduction of 3% in F_G regarding its optimal value was included. The obtained results for both control strategies are shown in Figures 11 and 12. In addition, the error between the traded and produced P_E (see control objective θ_5) for each one of the controllers tested is presented in Figure 13.

Based on the results in Figures 11 and 12, it is possible to observe that even when the proposed EMPC₁ takes into account fewer decision variables than the EMPC₂, both the optimal sequence \mathbf{V} as well as the CHP system outputs have similar behavior in both cases. However, the main difference between both approaches could be better appreciated in Figure 14, where the computational time spent at each iteration is shown for both controllers. According to these results, it is possible to conclude that even when fewer decision variables are considered by using EMPC₁, the proposed approach is a suitable strategy

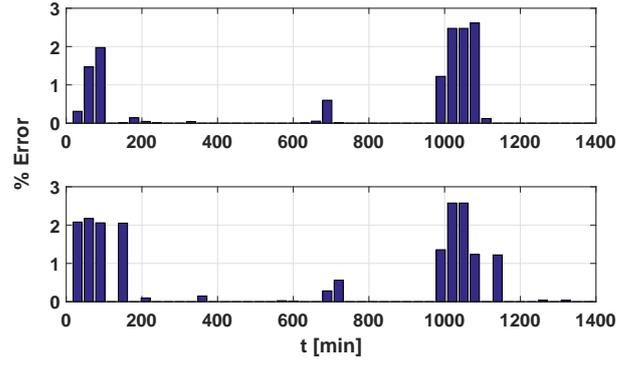


Figure 13: Comparison between P_E and P_{tra} by using EMPC₁ and EMPC₂.

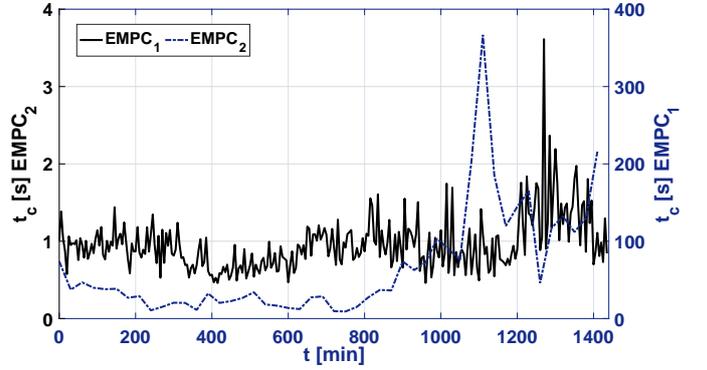


Figure 14: Computational time spent by iteration using EMPC₁ and EMPC₂.

to reduce the computational load without loss of optimality. Besides, it worth noting that, according to results in Figure 14, the EMPC using a constant time-step size is not suitable for its implementation in real time since the time required to solve the optimization problem is quite large.

It should be noted that, since EMPC₁ does a prediction of P_{tra} for the next 15 minutes and EMPC₂ only can do the prediction each 30 minutes due to the value of t_s , the obtained results using the proposed EMPC₁ was organized to compute the differences between P_E and P_{tra} at each 30 minutes. Thus, according to results in Figure 13 and Table 4, differences between P_E and P_{tra} could be reduced up to 2% for some time intervals (30 minutes) if the proposed EMPC₁ is implemented. This behavior is given that by using EMPC₁ it is possible to define more decision variables in the near future, which brings more flexibility to the controller to be able to achieve the control objectives while satisfying the constraints. Besides, by using shorter t_s in the near future along H_p , the smaller execution time for the controller could be implemented than when EMPC₂ is used. From this fact, the real behavior of the CHP system can be updated more frequently and, based on this, either disturbances or non-desired behaviors can be properly managed by the controller. Finally, in Table 4 the operating costs, revenue, and total costs for a simulation day are presented, from which it is possible to see that both strategies achieve similar optimal points.

Table 4: Total costs for one operation day by using EMPC₁ and EMPC₂.

Approach	Operating costs	Revenue	Real costs
EMPC ₁	137.12 e.u.	5.90 e.u.	131.22 e.u.
EMPC ₂	132.12 e.u.	5.87 e.u.	131.25 e.u.

$$A = \begin{bmatrix} 0.7718 & 0.5131 & -0.2202 & 0.1384 \\ 0.1625 & 0.9777 & 0.3117 & 0.2257 \\ 0.224 & 0.1613 & 0.691 & -4.852 \times 10^{-4} \\ -0.4279 & -0.4365 & -0.08312 & 0.6417 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.2625 & 0.0002352 & -0.005585 \\ -0.04581 & -9.191e-05 & -0.003041 \\ 0.1334 & 0.0003315 & -0.01392 \\ -0.1721 & -0.0002933 & 0.02044 \end{bmatrix},$$

$$C = \begin{bmatrix} 1720 & -8822 & -2238 & -2876 \\ 46.48 & -36.18 & -0.08659 & -3.471 \end{bmatrix},$$

and $D = 0$.

7. Conclusions

A predictive-like controller has been designed considering a non-constant time-step size along the prediction horizon and a soft constraint to include the energy-market constraints with the aim to reduce the computational time and minimize the total costs during the operation of the cogeneration plants. In this sense, a lower time-step size has been defined for near instants with the aim to include more decision variables and be able to satisfy the electric power traded and reported to the electric company. Besides, by including energy-market constraints into the optimization problem, the agreements of trading electric power with the electric companies could exist and allow reducing operating costs avoiding the economical penalties and getting revenues for the sale of the electric power.

Based on the obtained results, a lower computational cost for long prediction horizons with a great number of decision variables was achieved without loss of the optimality. In addition, significant cost reductions per day can be achieved while the economic penalties by the differences between the generated electric power and the traded electric power are avoided. In this regard, taking into account that the proposed control strategy is fast enough regarding the energy price updating, a stage of co-design for the implementation of the proposed control strategy in the real system could be developed.

On the other hand, the proposed control strategy could be tested for more complex systems, e.g., a microgrid with two or more different CHP systems and only one TES unit, with the aim to validate its performance. Besides, in order to test the performance of the proposed control strategy and the technique of model identification, scenarios in which disturbances and noise in both the model and measurements take place should be considered.

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APPENDIX

The matrices obtained by using the System Identification Toolbox™ provided by Matlab®, and corresponding to the discrete-time identified state-space model, are the following:

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