Distributed Model Predictive Control applied to a VAV based HVAC system based on Sensitivity Analysis

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Abstract: This paper proposes a distributed model predictive control strategy for a HVAC system that relies on its decomposition into subsystems based on the sensitivity analysis. An economic model predictive controller is implemented for every subsystem to optimize the operational cost of the building without compromising the thermal comfort of the occupants. Also, this work demonstrates the coordination strategy between the subsystem controllers using the sensitivity analysis approach. A discussion about the coordination strategy with the convergence property is provided. Finally, the proposed approach is illustrated using the benchmark building that considers the weather of Nancy, France.

Keywords: VAV type HVAC system, Distributed Mode Predictive Control, Energy Optimization

1. INTRODUCTION

Since last three decades, the energy crisis has been certainly one of the strong motivation in the changes of the HVAC industry towards more energy-efficient buildings without compromising the comfort. As energy requirement and fuel consumption of heating, ventilation and air-conditioning (HVAC) systems have a direct impact on the operational cost of a building as well as an impact on the environment. For this reason, building energy management has become an important issue in many countries (EU, 2016). More sophisticated technological schemes are now being developed and implemented in the buildings to reduce energy consumption, as e.g., thermal storage, building energy management systems (BEMS), advanced direct digital control, variable-air-volume (VAV) systems, variable frequency drives, etc. In order to improve the performance in HVAC various control techniques are developed e.g. gain scheduling in PID controllers, optimal control, adaptive control, nonlinear control, neural and fuzzy control methods. In above all, model predictive control (MPC) is favored control method due to its obvious advantages of handling constraints and disturbances. The detailed analysis of the available HVAC control techniques are summarized in Afram and Janabi-Sharifi (2014).

In large non-residential and commercial buildings, the HVAC system must meet the varying needs of different spaces since different zones of a building may have different heating and cooling needs. Due to these reasons, distributed control strategies are becoming very popular (Lamoudi, 2012). These control strategies hold various advantages as multivariable interactions, scalability and isolation in case of occurrence of faults. The class of DMPC methods are based on the type of decomposition of a large scale system into subsystems and the type of coordination between subsystem controllers. In the survey by Afram and Janabi-Sharifi (2014) about DMPC methods applied to HVAC systems, various approaches are found in the literature in the context of the type of the decomposition of centralized MPC problem e.g. primal an dual decomposition as in P. Pflaum and M. Alamir (2014), Dantzing Wolfe and Bender's decomposition as in Petru-Daniel Morosan (2010). In spite of the developments in the DMPC methods, much work is still needed to be done regarding its application to the HVAC systems.

In this work, we propose: i) a method to decompose the HVAC building system into subsystems based on the sensitivity analysis and ii) the coordination strategy between the subsystem controllers considering the sensitivities of the neighboring subsystem controllers and the coupling information.

For the illustration of the proposed method, we consider a benchmark HVAC building system with VAV systems. We consider every zone is provided with a VAV box in which a damper manipulates the airflow of supply air with the constant temperature into the zones to maintain the thermal comfort. This supply air with constant temperature is provided by an Air Handling Unit (AHU). The complexity of the centralized control increases exponentially as the number of zones increases. Hence, we propose the DMPC approach to achieve the same performance as centralized

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control architecture without compromising the thermal comfort of the occupants.

The organization of this paper is as follows. Section 2 describes the details of the HVAC building system under consideration for the demonstration of the proposed DMPC approach. Section 3 provides a brief discussion of the decomposition method to partition a system into the subsystems based on the sensitivity analysis. In Section 4, the detailed control objectives are formulated. The proposed sensitivity based DMPC approach is explained in Section 5 to achieve the above control objectives. The proposed DMPC method is evaluated using the simulation platform for the benchmark HVAC building system in Section 6 including a comparative analysis. Finally, Section 7 concludes the paper and provides some paths for the future work.

2. HVAC SYSTEM OF THE BUILDING DESCRIPTION

In this section, we describe a benchmark school building which is used to demonstrate the proposed sensitivity based DMPC approach. The building has two floors with 8 zones having a total area $648m^2$. The cross sectional layout for the benchmark building is shown in Figure 1. This

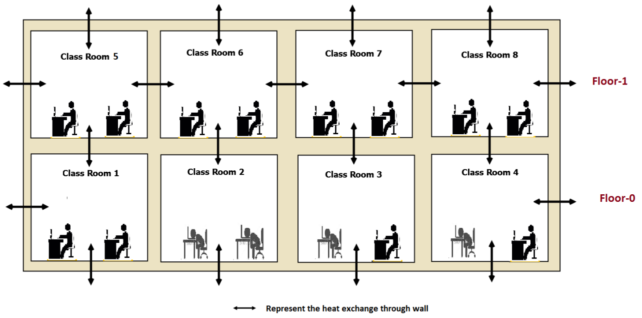

Fig. 1. Building Distribution: two floors with 4 classsrooms each

benchmark building is served by the VAV based HVAC system. Each zone has a VAV terminal, temperature sensor and a return air plenum. The VAV terminal provides supply air flow to each zone in order to maintain the thermal comfort which is recirculated to the AHU. AHU contains heating coil, mixer, and supply fan. Supply fan forces the supply air of constant temperature into the zones. The supply airflow rate at each zone is controlled such that the zone temperature is maintained in the thermal zones. Then, the supply air is recirculated to the AHU. In AHU, the fraction of recirculated air is combined with fresh air in the mixer. Then, the temperature of the air is increased in the heating coil which is an air-water heat exchanger. We consider that the hot water supply for the AHU unit is from production units as the heat pump or the boiler.

2.1 Thermal zone model

For each zone i, $(i = 1, ..., n)$ where $n = 8$ denotes the temperature of the zone by T_i, $\dot{m}_i$ mass flow rate at the output of the i-th VAV box and T_s is the supply

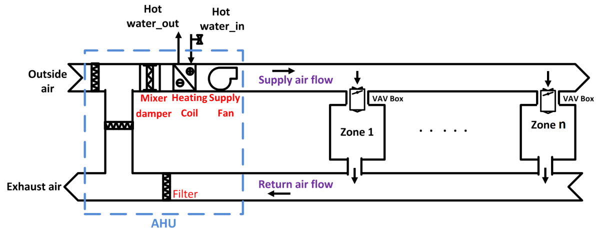

Fig. 2. VAV based HVAC system

air temperature. Then, the first law of thermodynamics applied to each zone is

$$C_i \frac{dT_i}{dt} = \dot{m}_i c_p (T_s - T_i) - \frac{1}{R_{ext_i}} (T_i - T_{oa}) - \sum_{j=1, j \neq i}^{n} \frac{1}{R_{ij}} (T_i - T_j) + q_i \qquad (1)$$

where C_i is the thermal capacitance of zone i, $R_{ij} = R_{ji}$ is the thermal resistance between zone i and zone j and R_{ext_i} is the thermal resistance between zone i and the exterior of the building. T_{oa} is the outside temperature and q_i is the heat flux due to occupancy and electronic devices. Linearized model from (1) is discretized using Euler method with a sampling period t_s and leads to written in general form as,

$$h_i(x_i, x_j, u_i) = a_i x_i + \sum_{j=1, j \neq i}^{n} a_{ij} x_j + b_i u_i + g_i w + q_i \qquad (2)$$

where state x_i is the zone temperature T_i, input u_i is the supply air flowrates $\dot{m}_i$ and w as the outside temperature.

3. SYSTEM DECOMPOSITION

The decomposition of the large scale system into the subsystems is one of the key problem addressed in the distributed control literature. We propose an approach based on the global sensitivity of the system motivated by Sobieski Sobieszczanski-Sobieski (1988). He suggested to obtain the system sensitivity equations to evaluate the internal couplings and system behavior related to variable changes. This approach has been used for distributing the computing task of mathematical model design into various engineering disciplines in the 90s, especially for the aircraft wing design problems. In this work, this notion is extended to decompose the large scale system into the subsystems. The proposed decomposition approach based on the sensitivity analysis is briefly explained in the following sections.

3.1 Sensitivity Matrix

The sensitivity equations are the partial derivatives of the system outputs with respect to the independent inputs. It is clear that if thermal balance for i-th zone as shown in (1) is simplified, it represents the change in zone temperature with respect to the inputs as supply flow rate, supply air temperature and temperatures of the neighboring zones. For example, the coefficients a_{ij} in (2) represents the sensitivity of the i-th zone temperature with respect to j-th zone temperature. The values of the coefficients b_i

represents the sensitivity of i-th temperature zone with respect to the i-th input (u_i). Note that inputs from the neighboring zones (u_j) will affect the the i-th zone temperature through j-th zone temperature (x_j). This will be accounted in the coefficient a_{ij}.

We write the thermal balance equations (2) for all the zones and represent them in matrix form as follows,

$$\mathcal{S}_{gs} = \begin{array}{c} \\ x_1 \\ \vdots \\ x_n \\ u_1 \\ \vdots \\ u_n \end{array} \begin{array}{c} \begin{array}{ccccc} h_1 & h_2 & \dots & h_{n-1} & h_n \end{array} \\ \begin{pmatrix} \frac{\partial h_1}{\partial x_1} & \frac{\partial h_2}{\partial x_1} & \cdots & \frac{\partial h_{n-1}}{\partial x_1} & \frac{\partial h_n}{\partial x_1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{\partial h_1}{\partial x_n} & \frac{\partial h_2}{\partial x_n} & \cdots & \frac{\partial h_{n-1}}{\partial x_n} & \frac{\partial h_n}{\partial x_n} \\ \frac{\partial h_1}{\partial u_1} & \frac{\partial h_2}{\partial u_1} & \cdots & \frac{\partial h_{n-1}}{\partial u_1} & \frac{\partial h_n}{\partial u_1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{\partial h_1}{\partial u_n} & \frac{\partial h_2}{\partial u_n} & \cdots & \frac{\partial h_{n-1}}{\partial u_n} & \frac{\partial h_n}{\partial u_n} \end{pmatrix} \end{array} \quad (3)$$

where the i-th column represents h_i denoting the thermal balance for i-th zone. The rows represent the variables with respect to which the sensitivity is calculated. This sensitivity matrix contains the information about the system couplings with the states and the inputs. The following section explains the methodology to exploit this information in the system decomposition. The off-diagonal coefficients in this block matrix represent the degree of the sensitivity of the state variables x $(x_1, \dots, x_n)$ with respect to other state variables and inputs u $(u_1, \dots, u_m)$. The basic idea behind the decomposition is to partition the matrix (3) into p block diagonal form. Every block will represent the group of zones representing a subsystem. The methodology of the matrix partition ensuring the minimal loss of information is explained in detail in the next section.

3.2 Partitioning based on sensitivity

The sensitivity matrix obtained in (3) is a large scale sparse matrix. There are various methods proposed in the literature to transform a sparse matrix into the block diagonal form (Golub and Loan, 1996; Pothen and Fan, 1990). In this work, we use the nested ϵ decomposition method (Siljak, 1991). This method is based on the graph theory and is very popular in the matrix decomposition literature. In this method, matrix coefficients that are less than ϵ are replaced by zeros. Then, the modified matrix is reordered to obtain a block diagonal form. Often, this procedure is carried out iteratively by augmenting ϵ such that $(\epsilon_k < \epsilon_{k+1})$ where k represents the iteration till the block diagonal form is achieved.

Let $\mathcal{S}_{gs}^{\epsilon_k}$ be the matrix after eliminating matrix elements less than ϵ_k at k-th interval. This matrix $\mathcal{S}_{gs}^{\epsilon_k}$ is permuted to obtain a diagonal form $\overline{\mathcal{S}}_{gs}^{\epsilon_k}$ using existing algorithms as e.g. reverse Cuthill-McKee algorithms (Golub and Loan, 1996).

To ensure minimal loss of the information in the modified sensitivity matrix $\overline{\mathcal{S}}_{gs}^{\epsilon_k}$,

$$\|\lambda_{ev}(\overline{\mathcal{S}}_{gs}) - \lambda_{ev}(\overline{\mathcal{S}}_{gs}^{\epsilon_k})\|^2 \leq \|\lambda_{ev}(\zeta)\|^2 \quad (4)$$

where $\overline{\mathcal{S}}_{gs}$ is the sensitivity matrix $\mathcal{S}_{gs}$ after applying the same permutation applied to the $\overline{\mathcal{S}}_{gs}^{\epsilon_k}$. λ_{ev} is used as the symbolic representation to denote eigenvalues of the matrix and ζ is the user defined tolerance matrix. The condition (4) should be verified for each iteration. The detailed procedure of the decomposition of sensitivity matrix into block diagonal form is stated in the Algorithm 1.

Thus, the sensitivity analysis is extended to obtain p subsystems. Note that these subsystems may share the states depending on the selection of the decomposition architecture. Overlapping diagonal blocks represent coupled subsystems through shared states. Contrary, non-overlapping diagonal blocks represent decoupled subsystem. To obtain, better results, it is possible to obtain the mixture of overlapping and non overlapping architectures. The proposed approach of DMPC is independent of the architecture of the decomposition due to the formulation discussed in next sections.

After applying Algorithm 1, we partition the building system into p groups containing highly coupled zones. Let the i-th group $\bar{h}_i$ contains n_i zones, satisfying $\sum_{i=1}^{p} n_i = n$ and $\sum_{i=1}^{p} \bar{h}_i = \sum_{i=1}^{n} h_i$.

Algorithm 1 Decomposition of global sensitivity matrix

Input Data: $\mathcal{S}_{gs}$,ϵ_0, ζ
Result : $\overline{\mathcal{S}}_{gs}$
Iterate : $k = 0$

(1) Replace $\mathcal{S}_{gs}(ij)$ by zero if $\mathcal{S}_{gs}(ij) < \epsilon_k$
(2) Permuting $\mathcal{S}_{gs}^{\epsilon_k}$ system using sparse reverse CM methods into the matrix $\overline{\mathcal{S}}_{gs}^{\epsilon_k}$
(3) Verify the condition $\|\lambda_{ev}(\overline{\mathcal{S}}_{gs}) - \lambda_{ev}(\overline{\mathcal{S}}_{gs}^{\epsilon_k})\|^2 \leq \|\lambda_{ev}(\zeta)\|^2$ is satisfied
(4) If matrix is still not close to the diagonal enough augment ϵ_k to ϵ_{k+1} and repeat step 1
(5) Otherwise, identify overlapping/nonoverlapping separable blocks from modified $\mathcal{S}_{gs}^{\epsilon_k}$ matrix

4. COST FUNCTION FORMULATION

The formulation of the cost function for the considered VAV based HVAC building systems include the following objectives (i) to minimize the economic operational cost, (ii) to maintain the thermal comfort in the zones and (iii) to generate smoother control signals by eliminating fluctuations to increase the actuator life-time. Detailed formulation to achieve mentioned goals are explained below.

4.1 Economic Cost function

Economic cost function in the proposed model predictive control refers to the total cost of the energy consumed by the building components, mainly by the supply fan and a heating coils in the AHUs. Let J be the total cost for a time interval $[t_0, t_f]$,

$$J_{i_e} = \int_{t_0}^{t_f} (J_h + J_{fan})\, dt \quad (5)$$

where J_h and J_{fan} are the costs due to energy consumed by the heating coil and the supply fan in the AHU:

(1) *Energy cost at the heating coil.* The power or heat transfer rate ($\dot{Q}_{coil}$) in the AHU required at the

heating coil to deliver an airflow at temperature T_s is directly obtained from the energy conservation law

$$\dot{Q}_{coil} = \sum_{i=1}^{n} \dot{m}_i c_p \left(T_s - T_{m_i}\right) \tag{6}$$

where T_{m_i} is temperature of air at the output of mixer. Then, the energy cost due to heating is simply given by

$$J_h = c_1 \dot{Q}_{coil_i} \tag{7}$$

where c_1 represents the related energy cost per kWh.

(2) *Energy cost delivered for the mass airflow.* The VAVs require a certain total mass airflow depending on each local (zone) heating load. This mass airflow is discharged by the power fan which is driven by a variable speed drive. The power fan characteristics for the AHU is given by a cubic law, that is,

$$\dot{W}_{fan} = \alpha \Big(\sum_{i=1}^{n} \dot{m}_i\Big)^3$$

With the above power characteristics, the cost the energy for a supply fan is as follows,

$$J_{fan} = c_2 \dot{W}_{fan_i} \tag{8}$$

where c_2 is corresponds to energy cost per kWh.

Thus, the total power demand from the AHU can be summarized from (5), (7) and (8). Note that J is a functional depending on the decision variables u, the state x and the disturbance d on $[t_0, t_f]$. In a discrete-time setting, the value of this integral (5) during sampling interval $[t_k, t_{k+1}]$ for any $k = 0, 1, ..., n$ is exactly given by $\int_{t_k}^{t_{k+1}} J_e d\tau = J_{ik}$. Then, we discretize J_e integral with Euler method using sampling time h. Hence the total economic cost for the building operation,

$$\ell_k^e(u(k)) = c_1 \sum_{i=1}^{n} u_i(k)^2 + c_2 \sum_{i=1}^{n} u_i(k) \tag{9}$$

4.2 Thermal comfort

To maintain the thermal comfort in the zones, the temperature should be controlled in the range of $[x^{min}, x^{max}]$. These bounds are relaxed to allow economic optimization,

$$-\zeta + x^{min} \leq x(k) \leq \zeta + x^{max} \tag{10}$$

where ζ is relaxation parameter with $0 \leq \zeta \leq 0.5$. Hence, the magnitude of this relaxation parameter ζ is considered as optimization variable by adding a penalty term in the optimization problem defined by,

$$\ell_k^{tc}(\zeta) = \zeta^2 \tag{11}$$

4.3 Elimination of fluctuations in the control signal

The above cost functions (9) and (11) considers the energy use and thermal comfort aspects. In addition, we introduce a term which indirectly addresses the maintenance cost. This is achieved by minimizing the variations in the control signal. Hence, the smooth control signals reduces the fatigue in the actuators, lowering the system maintenance cost. This term is a regularization term that is formulated as one norm over a variation of control signal shown below,

$$\ell_k^{re}(u(k)) = \lambda\{\|u(k) - u(k-1)\|_1\} \tag{12}$$

where $u_i(k-1)$ is the the control input implemented at previous instant and λ is the regularization parameter with $\lambda > 0$. For more details, readers are refered to Gallieri (2014) and Darure et al. (2016).

Now, the total cost for the k-th instant can be expressed as

$$J\left(u(k), x(k), \zeta\right) = \alpha_e \ell_k^e(u(k)) + \alpha_{tc} \ell_k^{tc}(\zeta) + \alpha_{re} \ell_k^{re}(u(k)) \tag{13}$$

where α_e, α_{tc} and α_{re} are the appropriate weights defined by the user.

5. SENSITIVITY BASED COOPERATION IN DMPC

In the proposed DMPC, the coordination of the subsystem controllers is based on the sensitivity of the control actions with respect to the neighboring subsystem information.

Let us consider that optimization problem associated to the centralized MPC can be expressed as follows,

$$\begin{aligned} \underset{U_k, X_k, \zeta}{\text{minimize}} \quad & J_N\left(U_k, X_k, \zeta\right) \\ \text{subject to} \quad & \\ & h(X_k, U_k) = 0 \\ & x^{min} \leq X_k \leq x^{max} \\ & u^{min} \leq U_k \leq u^{max} \\ & x\left(k|\, k\right) = x(k) \\ & \zeta \geq 0 \end{aligned} \tag{14}$$

where $U_k = \{u(k), \ldots, u(k+N-1)\}$ and $X_k = \{x(k+1), \ldots, x(k+N)\}$ are the sequences of predicted control inputs at time k. Also, $J_N\left(U_k, x_k, \zeta\right)$ is the cost function (13) over the prediction horizon N. The bounds on the input vector u i.e. on the supply airflow rate $[u^{min}, u^{max}]$ represent the damper limits in the VAV box. The bounds on states $[X^{min}, X^{max}]$ represent the soft bounds on the zone temperature to maintain thermal comfort.

For better understanding of the proposed approach let us rewrite (14) in simplified form as,

$$\begin{aligned} \mathcal{J} = \underset{z}{\text{minimize}} \quad & \phi(z) \\ \text{subject to} \quad & \\ & h(z) = 0 \\ & z^{min} \leq z \leq z^{max} \\ & z = [z_1, \ldots, z_{n+m}] \end{aligned} \tag{15}$$

where $z = (X(k)k, U(k), \zeta)$ and the function $\phi(z)$ corresponds to (J_N) the overall cost over the prediction horizon N.

Using the decomposition approach presented in the Section 3, we decompose the large scale system into the p subsystems by grouping the highly coupled states $(\hat{h}_{n1} ..., \hat{h}_{np})$. Let us formulate the sensitivity based optimization problem for the i-th subsystem,

$$\begin{aligned} \mathcal{J}_i = \underset{z_i}{\text{Minimize}} \quad & \phi(z_i, \bar{z}_j) + \left\{\frac{\partial \phi}{\partial z_i} + \sum_{\substack{k=1 \\ k \neq i}}^{p} \frac{\partial \hat{h}_k}{\partial z_i}\right\}(z_i - \bar{z}_i) \\ \text{subject to} \quad & \\ & \hat{h}_i(z_i) = 0 \\ & z_i^{min} \leq z_i \leq z_i^{max} \\ & z_i = [z_1, \ldots, z_{n_i}] \end{aligned} \tag{16}$$

where $\bar{z}_i$ and $\bar{z}_j$ are initial feasible values of the subsystems. Note, this formulation will be valid irrespective of separability property of the overall cost function ϕ.

As observed from this formulation, the cost function includes the terms representing sensitivity of i-th subsystem with respect to j-th subsystems.

From (15), the Lagrange function $\mathcal{L}$ is as follows,

$$\mathcal{L}(z) = \phi(z) + \sum_{k=1}^{n} \lambda_k h_k(z) \tag{17}$$

where λ_k are Lagrange multipliers. Now, the necessary condition of optimality in Boyd (2009) for the Lagrange function (17) at initial feasible point $(\bar{z}, \bar{\lambda})$ is

$$\nabla L|_{\bar{z}} = \frac{\partial \phi(z)}{\partial z}\Big|_{\bar{z}} + \lambda \frac{\partial h(z)}{\partial z}\Big|_{(\bar{z},\bar{\lambda})} \tag{18}$$

The sensitivity based DMPC formulation applied to (16) allows the cost function for i-th subsystem be rewritten as follows

$$\phi_i(z_i, \bar{z}_j) = \phi(z_i, \bar{z}_j) + \sum_{\substack{j=1 \\ j\neq i}}^{p} \Big\{ \frac{\partial \phi}{\partial z_i}\Big|_{\bar{z}_j} + \bar{\lambda}_j \frac{\partial \hat{h}_j}{\partial z_i}\Big|_{\bar{z}_j} \Big\} (z_i - \bar{z}_i) \tag{19}$$

Let the Lagrange function for i-th subsystem be,

$$\mathcal{L}_i(z_i) = \phi_i(z_i, \bar{z}_j) + \sum_{\substack{j=1 \\ j\neq i}}^{p} \bar{\lambda}_j \hat{h}_j(z) + \lambda_i \hat{h}_i \tag{20}$$

The necessary condition of the optimality of the i-th local subsystem MPC is given by,

$$\frac{\partial \phi(z_i, \bar{z}_j)}{\partial z_i} + \sum_{\substack{j=1 \\ j\neq i}}^{p} \bar{\lambda}_j \frac{\partial h_j(z_i, \bar{z}_j)}{\partial z_i} + \lambda_i \frac{\partial h_i(z_i, \bar{z}_j)}{\partial z_i} = 0 \;\; i \neq j$$

Now, necessary condition for optimization for all p subproblems,

$$\sum_{i=1}^{p} \sum_{\substack{j=1 \\ j\neq i}}^{p} \frac{\partial \phi(z_i, \bar{z}_j)}{\partial z_i} + \bar{\lambda}_j \frac{\partial h_j(z_i, \bar{z}_j)}{\partial z_i} + \lambda_i \frac{\partial h_i(z_i, \bar{z}_j)}{\partial z_i} = 0 \tag{21}$$

Note that the necessary condition for the centralized problem (18) and distributed problem (21) are equivalent. This indicates that the solution obtained by mean of the centralized architecture and sensitivity based distributed architecture are the same.

Algorithm 2 summarizes the method based on the problem formulation (14),

6. SIMULATION RESULTS

In the benchmark building introduced in Section 2, the occupants are present in the school in the working time i.e. from 08:00 to 18:00. The study is carried out during the winter season at Nancy in France. The plots of the heat flux due to occupancy and weather temperature over five workings days are shown in the Figure 3.

Thermal balance equations (2) are evaluated for every zone in the benchmark building. The data shown in Table (1)

Algorithm 2 Sensitivity based Distributed Model Predictive Algorithm

Initial Data: $\hat{h}$,x_k^0, u_k^0,u^{max},u^{max},x^{max},x^{max}, J

(1) Solve the problem (16) for all the subsystems at local level
(2) Implement the solution $u_i^*(k)$ to the i-th subsystem
(3) Obtain the measurements $x(k)$ from the subsystems
(4) Previous implemented control signals and measurements of states will be initial condition x_{k+1}^0, u_{k+1}^0 for $(k+1)$-th instant for each problem (16)
(5) Repeat step 1

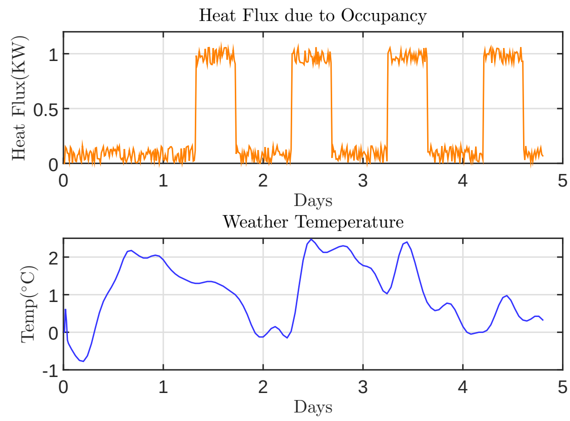

Fig. 3. Disturbances

C_i	$4.5\ kJ/s$	R_{ext}	$6\ W/°C$
R_{ij}	$18\ W/°C$	c_p	$1.005\ kJ/kg°C$
T_{oa}^0	$5\ °C$	T_s^0	$28\ °C$
$\dot{m}_i^0$	$0.192\ m^3/s$	q_i	$0.65\ kW$
T^{min}	$22\ °C$	T^{max}	$24\ °C$
$\dot{m}^{min}$	$0.192\ m^3/s$	$\dot{m}^{max}$	$0.42\ m^3/s$
T_i^0	$23\ °C$	N	$24\ h$

Table 1. Numerical data values used for benchmark building simulation

is used in simulating the case study school building (Tashtousha et al., 2005). The sensitivity matrix (3) is calculated and partitioned to obtain the subsystems as groups of zones. From the building layout presented in Figure 1, we obtain two groups $p = 2$ as $\{1, 2, 3, 4\}$ and $\{5, 6, 7, 8\}$. The number of groups or partitions are decided by the user. Applying Algorithm 2, sensitivity based DMPC controller is simulated considering the obtained subsystems. To represent the performance of the implemented architecture, the temperature response and corresponding supply airflow for zone 1 are shown in Figure 4 and Figure 5, respectively. Note that the thermal range and actuators limits can be different and is convenient to define the proposed distributed architecture.

According to the convergence analysis presented in Section 5, the performance of the sensitivity based DMPC is equivalent to the CMPC framework. This is verified from the temperature and supply airflow behavior of the respective control architectures. We also compare the control performance with decentralized MPC architecture (Siljak, 1991). In decentralized MPC, subsystem controllers operate independent of the state of the neighboring subsystems i.e. without any coordination or data exchange between

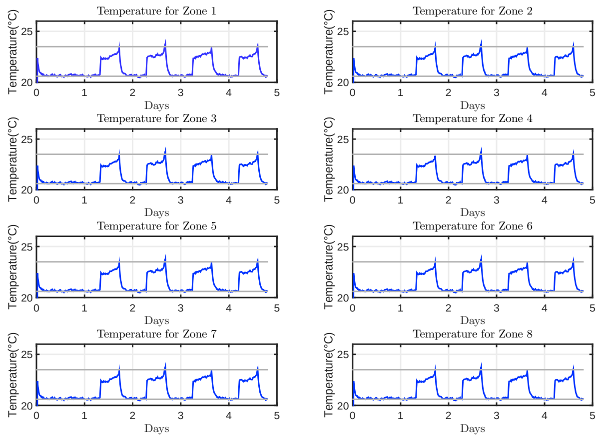

Fig. 4. Temperatures response for all the zones

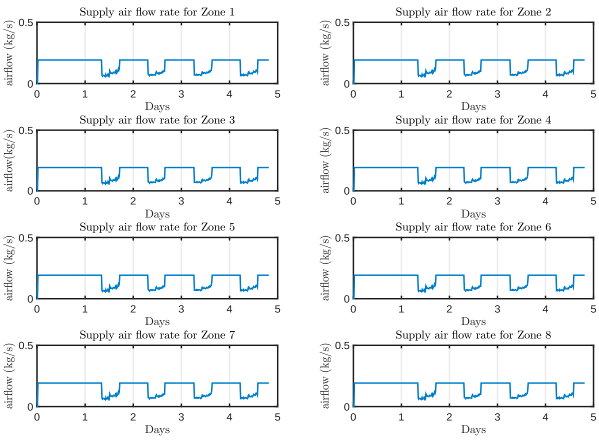

Fig. 5. Supply airflow rates for all the zones

the controllers. Also, the dynamics of the subsystems in decentralized control architecture completely ignores the coupling with the neighboring subsystems.

Figure 4 and the Figure 5 compares the performance of the all the control architectures. To support the previous conclusions, we also compare the energy consumed by the benchmark HVAC building over the five working days in the Figure 6. As, in the building system, the coupling between the zones are effective and if ignored, it results in consuming more energy and poor control performance.

7. CONCLUSION

In this work, we propose an approach that addresses the two stages of DMPC for the VAV based HVAC building system as i) the decomposition of the building system into subsystems and ii) the coordination between obtained subsystem controllers. Both objectives are based on the basic notion of sensitivity analysis. The proposed criterion in the method of decomposing the system into subsystems ensures minimum loss of the information of system dynamics. The method of sensitivity based coordination among the subsystem controllers is discussed in detail. The convergence analysis of the proposed method suggests

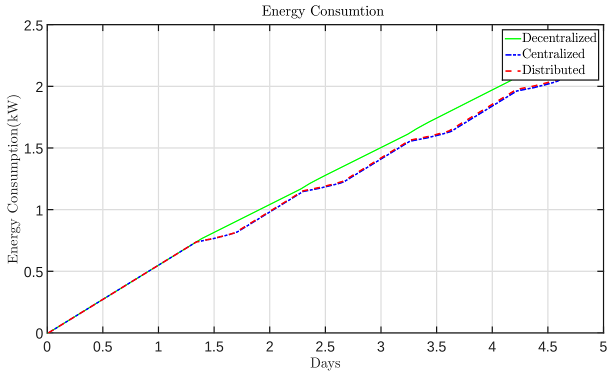

Fig. 6. Energy consumption

that the performance of the sensitivity based DMPC is equivalent to the CMPC performance. The decomposition and coordination strategies are demonstrated on the VAV based HVAC building.

The method can be adapted to different systems regardless of the separability of the cost function and the couplings between the subsystems. Future work will be focused for possible extension for the nonlinear systems and other types of HVAC buildings applications.

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