Output-Feedback Model Predictive Control of a Pasteurization Pilot Plant based on an LPV model

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Abstract. This paper presents a model predictive control (MPC) of a pasteurization pilot plant based on an LPV model. Since not all the states are measured, an observer is also designed, which allows implementing an output-feedback MPC scheme. However, the model of the plant is not completely observable when augmented with the disturbance models. In order to solve this problem, the following strategies are used: (i) the whole system is decoupled into two subsystems, (ii) an inner state-feedback controller is implemented into the MPC control scheme. A real-time example based on the pasteurization pilot plant is simulated as a case study for testing the behavior of the approaches.

1. Introduction

One of the important food preservation techniques is pasteurization, which is widely used in food industries. The pasteurization process implies applying heat to some products such as milk, cream, beer and others at a specified temperature for a specified period of time. This process is distributed into three sections that are heating, cooling and regeneration sections. The most important section is heating, which involves heat exchanger equipment to heat up the temperature of the product at the desired setpoint, and then maintaining this temperature during a constant time [11].

Among many pasteurization approaches, the High-Temperature Short-Time (HTST) approach is generally accepted as the industry standard [2]. In this process, the time temperature compound can change depending on the product and some of its properties as viscosity, fat percentage, solid residues, etc. Controlling and maintaining the temperature of the process is an important key in pasteurization. Accordingly, a suitable control system to control product temperature needs to be designed for keeping the desired product quality. The necessity of a significant control for the process arises from the saving in energy, product and time if an accurate tracking of the setpoint is performed, and if disturbance influences are rapidly and productively rejected.

Recently, model-based predictive control (MPC) technology gained its industrial position in the process industry [13], because of its ability to deal with multivariable control, delays and constraints on system variables [5]. The basic idea behind MPC is to repeatedly solve an optimal control problem on-line to find an optimal input to the controlled system. Nonetheless, continuous technological advances during the last years make possible the real-time implementation of the MPC controllers based on more complex and large-scale dynamical models. Therefore, several possibilities arise when implementing real-time predictive controllers for industrial processes.

Furthermore, a pasteurization system includes typical behaviors of relevant processes, such as complex dynamical models with nonlinearities, which imply important challenges when a suitable controller should be designed [14]. In the literature, there are different modeling approaches and controller strategies for pasteurization systems. In [6], the complete process model leads to a multivariable first order with pure delay transfer functions with variable parameters after decomposing the system into functional subsystems. Other models from the whole system and/or some subsystems are obtained in [1, 11]. Regarding its control, in [7] a Dynamic Matrix Control (DMC) is designed and implemented by using the system models proposed in [6]. In [11], it is proposed a scheme based on PID controllers with different tuning methods. The regulation of both water and milk temperatures by using MPC is reported in [12], where transient behaviors have been suitably handled with respect to other control techniques such as cascade generic model control (GMC) strategy. A recent study by [14] involved three different control topologies based on MPC that aim to the minimization of the energy consumption. However, after revising the literature the pasteurization plant have never been applied controller based on Linear Parameters Varying (LPV) models of the pasteurization plants. In addition, theses studies used MPC controller with full knowledge of the state, whereas, in many control problems, not all the states can be measured exactly. In this situation, the output-feedback MPC is more practical than the state-feedback one. Thus, an observer can be employed for estimating the states if the model of the system has the observability condition. One of the main problems that can be found for designing a state observer and estimating the states is to have an unobservable model. In this paper, some states of the system can not be measured exactly, therefore an observer is needed but the system model including disturbances has observability problem.

This paper proposes the design and simulation of an observer-based output-feedback MPC controller to regulate the temperature of the output product in a small-scale pasteurization plant, taking into account the complete model from the pasteurization system has unobservable states. The LPV model of the pasteurization plant is obtained from experimental data according to [7]. Then, the identified models obtained as transfer functions are suitably formulated by their equivalent controllable realizations in state-space with varying parameters. Owing to this, the main contribution of this paper is not only the suitable design of an output-feedback MPC controller with a state observer based on an LPV model of the pasteurization plant in order to satisfy the control objectives fulfilling system constraints, but also to address the observability problem of the pasteurization system when considering the disturbance model. Moreover, in this paper, the MPC controller is computed by using the LPV model during the simulation scenario and Linear Time Varying (LTI) along the prediction horizon.

The remainder of the paper is organized as follows. In Section 2, the pasteurization plant and pasteurization process are described. Besides, the mathematical models and statement of the control problem are presented. In Section 3, approaches for solving problems described in Section 3 are proposed. Section 4 presents and discusses the results. Finally, in Section 5, the conclusions of this work are drawn and some research lines for future work are proposed.

2. PROBLEM STATEMENT

2.1. System Description

The considered pasteurization process is the small-scale plant PCT-23 MKII, manufactured by Armfield (UK) [3]. This laboratory plant is the small version (1.2m, 0.6m, 0.6m) of several real-time industrial pasteurization process. The High-Temperature Short-Time (HTST) approach is generally accepted as the industry standard for the pasteurization process [6]. In this process, the goal is to heat and preserve the product, which is typically a liquid, at a predetermined temperature for a minimum time. This is achieved by circulating the heated liquid through a holding tube that delays the product stream [14]. Throughout the pasteurization process, the liquid is pumped at a predetermined flow rate from one of two storage tanks to the heat exchanger indirectly. The water heat is transferred to the product inside the first phase of the heat exchanger, which is called regenerator. The raw product is heated to an intermediate temperature by using missed energy from the pasteurized product. Then, this product is heated from that middle temperature to the full pasteurization temperature, in the second phase, while using a hot-water flow (F_h) coming from a closed circuit with a heater. The temperature (T_{past}) at the output of the holding tube is used to monitor the product temperature after the pasteurization process. Finally, the product is cooled in the third phase of the heat exchanger, where remaining heat is removed to the inlet product. This last phase does not add anything new to the model, and so it was not considered in this paper. In summary, from a control point of view, the pasteurization plant can be seen as a multiple-input and multiple-output (MIMO) system with electric heater power, P, and hot/cold flow ratio ($R = Fh/Fc$) as inputs, and temperatures water heater T_{ow}, and T_{past} as outputs. In addition, T_{iw} and T_{ic} are disturbances of the system, where T_{iw} is the input of water heater and T_{ic} is the cold temperature input of the heat exchanger.

2.2. Mathematical Model

The block diagram of the model used in this article has been established in [7], where the hot /cold water flow ratio ($R = F_h/F_c$) was modified to make possible its control. The complete scheme of plant is shown in Figure 1.

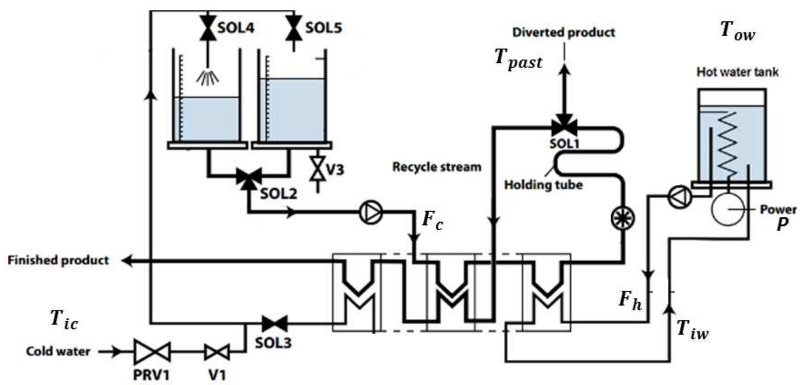


Figure 1. Plant scheme.

2.2.1. Hot water tank model: The heater subsystem is an electrically heated reservoir that is covered in order to minimize heat losses. In the hot water tank, the water is warmed by means of the power resistor, whereas a water pump with an upper limit of 700 ml/min moves the heated water. This flow, defined as hot flow F_h, transfers heat to the pasteurization product in a heat exchanger, before returning it to the hot water tank. Generally, this hot flow changes along the time in order to keep the pasteurization process. According to the dotted red box in Figure 2, the input temperature T_{iw} and the input power P are considered as two inputs for the hot water tank subsystem, while the temperature of the water heater T_{ow} is the system output.

2.2.2. Heat exchanger model: This section includes two effective phases, the former is based on the regeneration phase and the lattes is related to the heating phase. The heating phase (second phase) is more complex than the regeneration phase (first phase), also the first phase is obtained as a simplification of the second one, hence, the heating phase is studied first.

Heating phase (*second phase*): The block diagram obtained for the heat exchanger is shown in Figure 2 (green box). As reported in [7], the transfer functions related to

$$G_{ij} = \frac{K_{ij}(R)}{1+\tau_{ij}(F_c, F_h)s} e^{-\theta_y(F_c,F_h)s}. \tag{1}$$

The transfer functions G_{11} and G_{21} generally correspond to the hot water flow, while G_{12} and G_{22} describe the variations caused by the cold water flow. The pasteurization time (or holding time) is fixed

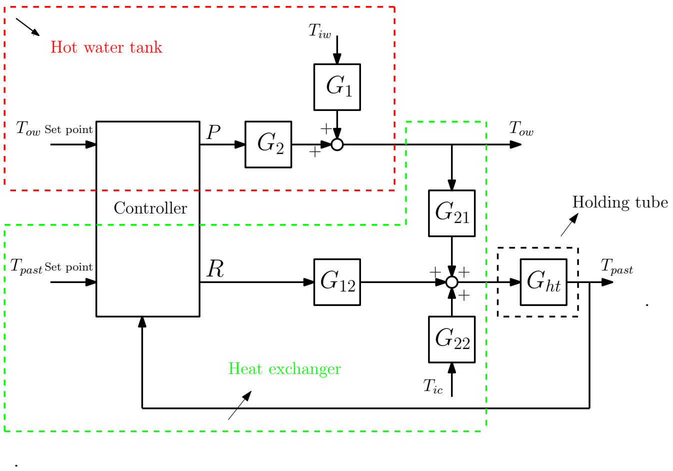


Figure 2. Block diagram of the control loop.

during the real pasteurization process. Hence, the cold water flow must be kept as a constant value. Therefore, G_{12} and G_{22} will show fixed dynamics, but with varying gain, since the static gain depends on R according to (1). Note that the parameters of (1) were obtained experimentally in [7].

Regeneration phase $(first\ phase)$: In this phase of the heat exchanger, the cold water and the hot water flows are the same, thus the relation $R = F_h/F_c$ is constant and of unitary value.

2.2.3. Holding tube model The model of the holding tube is not purely a transport delay because there were some heat losses. The holding tube can be modeled as a single-input single-output system. According to [7], the estimation parameters for transfer function G_{ht} (see inside dotted black line in Figure 2) are considered as static gain $K_{ht} = 0.91$ and time constant $\tau_{ht} = 21$s.

Accordingly, identified models obtained as transfer functions are suitably stated by their equivalent controllable realizations in stat-space, with varying parameters according to the hot water flow, F_h, as exogenous input and the hot/cold water flow ratio, R, as the input of the system. As discussed before, the state-space model of this plant has four inputs: P and R are the manipulated inputs, while T_{iw} and T_{ic} are disturbances. Moreover, T_{ow} and T_{past} the temperature of the hot water tank and the pasteurization temperature of the process are the first and second outputs of this system, respectively. Therefore, the continuous time state-space model of the pasteurization plant can be represented as follows:

$$
\begin{aligned}
x(k+1) = &\begin{bmatrix}
\frac{-1}{\tau_1(F_h(t))} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-1}{\tau_2(F_h(t))} & 0 & 0 & 0 & 0 \\
\frac{K_{21}(R(t))}{\tau_{21}(R,(t))} & \frac{K_{21}(R(t))}{\tau_{21}(R(t))} & \frac{-1}{\tau_{21}(R(t))} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{-1}{\tau_{12}(R(t))} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{-1}{\tau_{22}(R(t))} & 0 \\
0 & 0 & \frac{K_{ht}}{\tau_{ht}} & \frac{K_{ht}}{\tau_{ht}} & \frac{K_{ht}}{\tau_{ht}} & \frac{-1}{\tau_{ht}}
\end{bmatrix} x(k) \\
&+ \begin{bmatrix}
0 & 0 \\
\frac{K_2(F_h(t))}{\tau_2(F_h(t))} & 0 \\
0 & 0 \\
0 & \frac{K_{12}(R(t))}{\tau_{12}(R(t))} \\
0 & 0 \\
0 & 0
\end{bmatrix} u(k) + \begin{bmatrix}
\frac{K_1(F_h(t))}{\tau_1(F_h(t))} & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & \frac{K_{22}(R(t))}{\tau_{22}(R(t))} \\
0 & 0
\end{bmatrix} w(k),
\end{aligned}
$$

$$y(k) = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} x(k), \tag{2}$$

where K is a static gain and τ is the time constant of the transfer functions of the subsystems. In addition, the variable $x \in \mathbb{R}^{n_x}$ denotes the state vector of the plant, $u \in \mathbb{R}^{n_u}$ the control input vector, $w \in \mathbb{R}^{n_d}$ denotes the disturbance input vector and $y \in \mathbb{R}^{n_y}$ is the output vector. In this paper, the discretized state-space model of the complete system (2) with sampling time $T_s = 4$s is used.

2.3. Control Problem

As previously mentioned, in the pasteurization process there are two control objectives to be achieved. First, the raw product must be retained in the holding tube during the required pasteurization time and second the pasteurization temperature must be reached and maintained as close as possible to the set-point values. Consider the general state-space representation for discrete-time LPV systems

$$x(k+1) = A(\theta(k))x(k) + B(\theta(k))u(k) + E(\theta(k))w(k), \tag{3a}$$

$$y(k) = C(\theta(k))x(k), \tag{3b}$$

where the discrete time is denoted by $k \in \mathbb{Z}$, the system matrices $A(\theta(k)) \in \mathbb{R}^{n_x \times n_x}$, $B(\theta(k)) \in \mathbb{R}^{n_x \times n_u}$, $E(\theta(k)) \in \mathbb{R}^{n_x \times n_d}$ and $C(\theta(k)) \in \mathbb{R}^{n_y \times n_x}$ are assumed to depend linearly on the parameter vector $\theta(k) := [\theta_{1,k}, \theta_{2,k}, ..., \theta_{N,k}]^T \in \mathbb{R}^N$, which belongs to a convex polytope Θ defined by

$$\Theta := \left\{ \theta(k) \in \mathbb{R}^N | \sum_{j=1}^{N} \theta_{j,k} = 1, \, \theta_{j,k} \geq 0 \right\}, \tag{4}$$

where N is the number of vertices. Clearly, as $\theta(k)$ varies inside the convex polytope Θ, the matrices of the system (3) vary inside a corresponding polytope Ψ, which is defined by the convex hull (Co) of N local matrix vertices $[A_i, B_i, E_i, C_i]$, $i \in [1, .., N]$,

$$\Psi := Co\left\{ \begin{bmatrix} A_1 & B_1 & E_1 & C_1 \end{bmatrix}, \begin{bmatrix} A_2 & B_2 & E_2 & C_2 \end{bmatrix}, ..., \begin{bmatrix} A_N & B_N & E_N & C_N \end{bmatrix} \right\}, \tag{5}$$

and the matrices of the system (3) can be rewritten as

$$A(\theta(k)) = \sum_{j=1}^{N} \theta_{j,k} A_j, \quad B(\theta(k)) = \sum_{j=1}^{N} \theta_{j,k} B_j, \quad E(\theta(k)) = \sum_{j=1}^{N} \theta_{j,k} E_j, \quad C(\theta(k)) = \sum_{j=1}^{N} \theta_{j,k} C_j. \tag{6}$$

Moreover, the hot water flow F_h, and hot/cold water flow ratio R are the scheduled variables of the pasteurization model, where F_h and R are exogenous input and endogenous input of the system, respectively. The process of pasteurization changes so fast. Therefore, the signal at the current time step is equal to the previous step and next step ahead. In this situation, the equal scheduled variables of models are assumed along prediction horizon N_p in the LPV model. Thus, $\theta_{(k|k)} \simeq \theta_{(k+1|k)} \simeq \theta_{(k+2|k)} \simeq ... \simeq \theta_{(k+N_p-1|k)}$ can be considered in a short N_p. Based on the fact that one of the scheduled varietals is the manipulated input of the system, the following condition can be written in a few N_p:

$$u_{(k|k)} \simeq u_{(k+1|k)} \simeq u_{(k+2|k)} \simeq ... \simeq u_{(k+N_p-1|k)},$$

it means that current input $u_{k|k}$ can be used for the next system matrix $A(\theta_{k+1|k})$. Consequently, the model of pasteurization plant during the simulation scenario is considered as an LPV model. In addition, the LTI model is used along N_p into the MPC controller. Taking into account the control objectives described before, an MPC controller can be achieved for the pasteurization system with the following cost function:

$$\min_u J(k) = \sum_{i=0}^{N_p-1} ||Cx(k+i|k) - r(k+i+1|k)||_{W_1}^p + ||u(k+i|k)||_{W_2}^p, \tag{7a}$$

subject to

$$x(k+i+1) = A(\theta(k+i))x(k+i) + B(\theta(k+i))u(k+i) + E(\theta(k+i))w(k+i), \quad \forall i \in \{0,...,N_p-1\} \tag{7b}$$

$$y(k+i) = Cx(k+i), \quad \forall i \in \{0,...,N_p\} \tag{7c}$$

$$\underline{x} \le x(k+i) \le \overline{x}, \quad \forall i \in \{0,...,N_p\} \tag{7d}$$

$$\underline{u} \le u(k+i) \le \overline{u}, \quad \forall i \in \{0,...,N_p-1\} \tag{7e}$$

$$\theta(k+i) \in \Theta, \quad \forall i \in \{0,...,N_p\} \tag{7f}$$

where p denotes the norm used, also $W_1 \in \mathbb{R}^{n_x \times n_x}$ and $W_2 \in \mathbb{R}^{n_u \times n_u}$ are positive definite weighting matrices needed to prioritize the different control goals within the multi-objective cost function for state and the control input, respectively. The vector $r(k)$ is the reference. Vectors $\underline{x}$ and $\overline{x}$ determine the minimum and maximum possible state values of the system, respectively. Also $\underline{u}$ and $\overline{u}$ determine the minimum and maximum possible control inputs, respectively. To distinguish actual state and predicted trajectories, in what follows $k+i|k$ denotes future values at time $k+i$ predicted at time k. According to the model presented in this paper, not all states of the system can be measured while only the output $y(k)$ is available for measurement. Thus, an observer to estimate the system state is required for closing the feedback control loop. However, in (2), the pair (A,C) is not observable because of the inclusion of the disturbance models and it has caused the state related to the disturbance input generates an observability problem for the complete model. The following sections present and discuss two specific approaches for solving this problem.

3. PROPOSED APPROACH

3.1. Problem formulation

The pasteurization plant described in Section 2 can be controlled with a holistic approach by means of one single MPC controller that directly operates the actuators from the states measurement. A completed model of the pasteurization plant is needed for this approach. In order to design the corresponding MPC, the constraints and cost function in (7) should be defined. As already discussed, some states of the system can not be measured exactly and hence an observer should be employed to estimate the states. In this paper, the topology of the observer is based on the classical Luenberger structure [9]. The observer equations can be written as

$$\hat{x}(k+1) = A(\theta(k))\hat{x}(k) + B(\theta(k))u(k) + L(\theta(k))(y(k) - \hat{y}(k)) + E(\theta(k))w(k), \tag{8a}$$

$$\hat{y}(k) = C(\theta(k))\hat{x}(k), \tag{8b}$$

where $\hat{x} \in \mathbb{R}^{n_x}$ is the current observer state, $u \in \mathbb{R}^{n_u}$ is the current control input, $\hat{x}(k+1)$ is the successor state of the observer system, $\hat{y} \in \mathbb{R}^{n_y}$ is the current observer output and $L \in \mathbb{R}^{n_x \times n_y}$ denotes the observer gain. The design of the LPV observer can be achieved by considering the LMI pole placement technique [4], which allows to locate the poles of the observer in a subregion of the unit circle using an LMI region [10]. The motivation for seeking pole clustering in specific regions inside the unitary circle is to obtain fast observer dynamics for all considered operating points.

As mentioned in Section 2.3, the considered model in (2) becomes unobservable when the disturbance is added into the matrix system. In the following, two ways of solving this issue will be considered.

3.2. Decouple the complete system into two separate subsystems

In particular, while the complete model of pasteurization plant is unobservable, it becomes possible that the decomposed system becomes observable. In this way, the complete system can be decomposed into two separate subsystems: (i) hot water tank, (ii) heat exchanger and holding tube. Then, the two subsystems become observable. In this case, two independent controllers are needed. The first control loop (see inside the dotted black line in Figure 3) is dedicated to control T_{past} with the controller output R and considering the other inputs to the heat exchanger as measurable perturbations. The second controller will maintain T_{ow} constant in spite of changes both in R (and therefore in F_h) and in T_{iw}, with the power P as the controller output (see inside dotted red line in Figure 3).

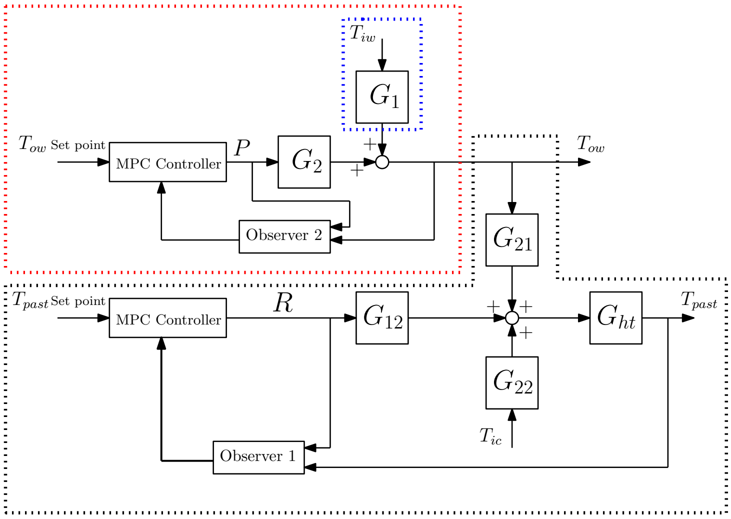


Figure 3. Scheme of split the complete system into two separate subsystems.

3.2.1. Hot water tank: According to Section 2, the hot water tank transfer function presents as dependent on the exogenous variable F_h, while the input temperature of water heater T_{iw} and power P are disturbance input and manipulated input of the hot water tank, respectively. Furthermore, the state-space model of this subsystem can be represent as

$$x_{hot}(k+1) = \begin{bmatrix} \frac{-1}{\tau_1(F_h(t))} & 0 \\ 0 & \frac{-1}{\tau_2(F_h(t))} \end{bmatrix} x_{hot}(k) + \begin{bmatrix} 0 & \frac{K_1(F_h(t))}{\tau_1(F_h(t))} \\ \frac{K_2(F_h(t))}{\tau_2(F_h(t))} & 0 \end{bmatrix} u_{hot}(k), \tag{9a}$$

$$y_{hot}(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} x_{hot}(k), \tag{9b}$$

where x_{hot}, u_{hot} and y_{hot} are the states, input and output of the hot water tank subsystem, respectively. Note that the pair (A,C) is not observable based on the existence of the disturbance in this subsystem. In this case, to avoid this problem, one option is to consider the unobservable part (see inside dotted blue line in Figure 3) of the system as a measurement disturbance. Therefore, the model (9) can be reduced by removing the unobservable part dynamic (blue line in Figure 3). Then, the evaluation of the disturbance in the future can be anticipated. Thus, the state-space of hot water model can be rewritten as

$$x_{hot}(k+1) = A(\theta(k))x(k) + B(\theta(k))u(k), \tag{10a}$$

$$y_{hot}(k) = C(\theta(k))x(k) + y_{unobs}(k), \tag{10b}$$

and

$$x_{unobs}(k+1) = A_{unobs}(\theta(k))x_{unobs}(k) + B_{unobs}(\theta(k))u(k), \tag{11a}$$

$$y_{unobs} = C_{unobs}(\theta(k))x_{unobs}(k), \tag{11b}$$

where x_{unobs} and y_{unobs} are states and output of the unobservable part dynamic of the subsystem, respectively. In addition, A_{unobs} and B_{unobs} are matrices system of this part. After all, the pair (A,C) is observable in the system (10). Therefore, the Luenberger observer is used for estimating the states of hot water tank subsystem.

3.2.2. Heat exchanger and Holding tube: As mentioned in Section 2, the transfer function of the heat exchanger second phase introduces a dependence on R, and the holding tube is a transfer function with constant values of both the static gain and time constant. In addition, the hot/cold flow R, hot water tank temperature $T_o w$ and input temperature of cold water, T_{ic} are inputs of this subsystem. Therefore, the state-space model of this part is represented as

$$x_{heat}(k+1) = \begin{bmatrix} \frac{K_{21}(R(t))}{\tau_{21}(R(t))} & 0 & 0 & 0 \\ 0 & \frac{K_{12}(R(t))}{\tau_{12}(R(t))} & 0 & 0 \\ 0 & 0 & \frac{K_{22}(R(t))}{\tau_{22}(R(t))} & 0 \\ \frac{K_{ht}}{\tau_{ht}} & \frac{K_{ht}}{\tau_{ht}} & \frac{K_{ht}}{\tau_{ht}} & \frac{-1}{\tau_{ht}} \end{bmatrix} x_{heat}(k) + \begin{bmatrix} 0 & \frac{K_{21}(R(t))}{\tau_{21}(R(t))} & 0 \\ \frac{K_{12}(R(t))}{\tau_{12}(R(t))} & 0 & 0 \\ 0 & 0 & \frac{K_{22}(R(t))}{\tau_{22}(R(t))} \\ 0 & 0 & 0 \end{bmatrix} u_{heat}(k) \quad (12a)$$

$$y_{heat}(k) = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} x_{heat}(k), \quad (12b)$$

where x_{heat}, u_{heat} and y_{heat} are the states, input and output of the second subsystem (heat exchanger and holding tube), respectively. According to observability rule, the state-space model in (12) is observable.

3.3. Complete system with local state-feedback controller

Another possibility to solve the unobservability of the complete model for pasteurization plant is to use another control loop inside the control scheme. The control scheme for this approach is presented in Figure 4.

In other words, making a closed-loop model of a complete dynamic model of the pasteurization plant by using the state-feedback controller before the MPC controller and in this case, the observability of the complete model of the pasteurization system is determined. It is assumed that a model of the plant is controllable, also as in the first approach, the control objective remains the same, forcing the controlled output to track the references whilst satisfying the constraints. As a matter of fact, just some states of the pasteurization system are measurable and these can be used into inner controller for determining the static gain (see Figure 4). Consider a state-feedback control law for inner control loop as

$$u^*(k) = K(\theta(k))x^*(k), \quad (13)$$

where x^* includes some measurable states and $K(\theta(k))$ is the scheduled control gain. A suitable gain for the whole LPV model is obtained via the weighted average of the control law designed for each vertex, i.e.,

$$K(\theta(k)) = \sum_{j=1}^{N} \theta_{j,k} K_j, \quad (14)$$

where $K_j \in \mathbb{R}^{n_u \times n_x}$ is a time-invariant feedback gain associated with the $j-th$ vertex system. Moreover, the stack of gains for the scheduling control law K_j should be found by solving an LQR problem [8] at each vertex. For the complete system in (2), by using the control law (13), the closed-loop model of system representation is written as

$$x(k+1) = A_{cl}(\theta(k))x(k), \quad (15a)$$

$$y(k) = C(\theta(k))x(k), \quad (15b)$$

where

$$A_{cl} = \sum_{i=1}^{N} \sum_{j=1}^{N} \theta_{i,k}\theta_{j,k}(A_i + B_i K_j), \quad (16)$$

and A_i, B_i are local matrix vertices with $[A_i, B_i]$, $i \in [1,..,N]$. In this case, analyzing the observability of the closed-loop model yields that the complete model with local controller is observable.

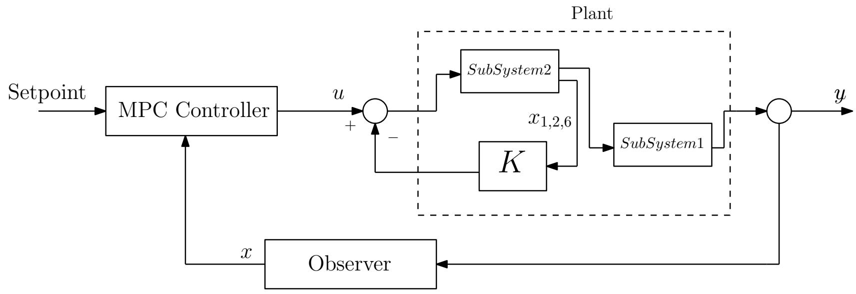


Figure 4. Control scheme of pasteurization process with inner state-feedback controller.

4. SIMULATION RESULTS

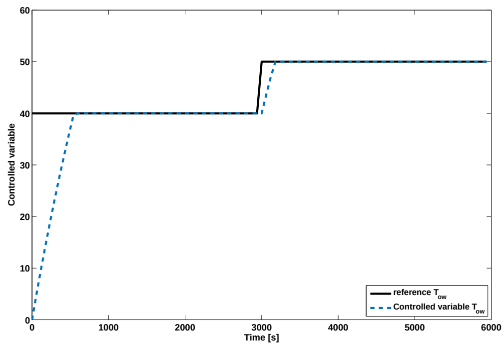
In this section, results of two of the proposed approaches are presented and analyzed in detail.

4.1. Scenarios

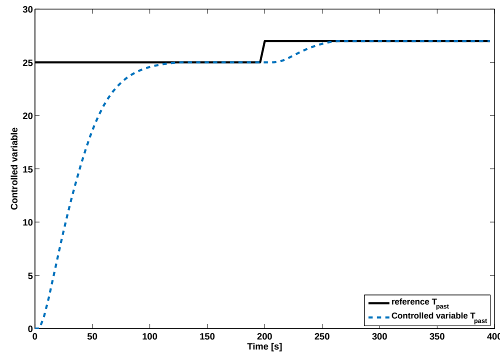
4.1.1. Scenario 1: As discussed in Section 3, the complete model of the pasteurization plant is separated in two parts: hot water tank and heat exchanger with holding tube. For this experiment, the water heater input temperature T_{iw} was maintained constant at $\simeq 30\,^{\circ}$C. Therefore, the behavior of the subsystem inside the dotted blue box in Figure 3 can be computed separately. Then, the value of the response is added to the subsystem as measurement disturbance in output. In addition, the LPV state observer is designed assuming that the eigenvalues are placed in an LMI region that is the intersection between the disk of radius $r = 0.6$ and center $q = 0$. The control objective of MPC controller for the hot water tank subsystem is temperature T_{ow} should be track the setpoint where their references change from $40\,^{\circ}$C to $50\,^{\circ}$C. The MPC controller has been implemented in simulation, where the weight matrices are $Q = 1$, $R = 0.01$. The prediction horizon is chosen $N_p = 5$ with $T_s = 1$ min. In second part of this approach, the pasteurization temperature T_{past} is the variable to be controlled and R will be used as the controller output. The first stage is supposed to be disconnected, so that the input temperature to the heat exchanger T_{ic} is the raw temperature and it will be maintained at $\simeq 20\,^{\circ}$C.

The control objective of MPC controller for the second subsystem is the temperature T_{past} and it should follow a predefined profile that is tracked the setpoint where their references change from $25\,^{\circ}$C to $27\,^{\circ}$C. The prediction horizon is chosen $N_p = 7$ where the weight matrices are $Q = 1$, $R = 0.04$ with $T_s = 4$s. Moreover, the state observer is designed by using the LMI method, where $r = 0.4$ and $q = 0$.

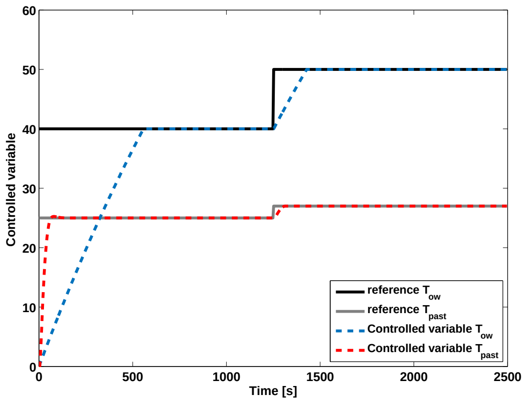
4.1.2. Scenario 2: The second approach involves the controller and observer of the complete system with inner state-feedback controller. The control gain for the state-feedback controller is computed by using the LQR method, while the measurable states as x_1, x_2 and x_6 are used also the control gain is computed at every iteration. Similar to the first scenario, the state observer is designed by LMI considering that the eigenvalues are located in an LMI region that is the intersection between the disk of radius $r = 0.6$ and $q = 0$. In order to improve the convergence of the observer, the following initial condition on the pasteurization plant and the observer states have been considered: $x(0) = \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T$, $\hat{x}(0) = \begin{bmatrix}5 & 5 & 5 & 5 & 5 & 5\end{bmatrix}^T$. Now, the MPC controller was implemented with the weighting matrices are $Q = 1$, $R = 1$ and the prediction horizon $N_p = 7$. Furthermore, the references of T_{past} and T_{ow} are considered in the same way as introduced in Scenario 1 when the T_{ic} and T_{iw} are maintained constant at $\simeq 30\,^{\circ}$C and $20\,^{\circ}$C as measurement input disturbances. In addition, the first input heating power of resistor P can take value in the range $[0, 2]$ KW and the range value for the second input, the hot/cold flow ratio R, is $[0.2, 2]$.



(a) T_{ow} with two MPC controller



(b) T_{past} with two MPC controller



(c) T_{ow} and T_{past} with a single MPC controller

Figure 5. Controlled temperature of pasteurization process with single MPC and two independent MPC.

4.2. Discussion

In order to test the behavior of the proposed control scheme regarding tracking and disturbance rejection, several simulations were carried out and the results obtained are presented in this part.

In Figure 5, the temperatures T_{ow} and T_{past} from the pasteurization plant under the MPC controller of the split model (two subsystems) and complete system without the observer are shown.

Figure 5a shows the output temperature of the hot water tank subsystem. In addition, the first reference point is assumed 40 °C from time instant 0s until 3000s and the second reference point is considered 50 °C from time instant 3000s until 6000s in Figure 5a. In Figure 5b, from time instant 0s until 200s, the first reference point is 25 °C and 27 °C is considered from time instant 200s until 400s for the second reference point in simulation of output temperature of heat exchanger subsystem. The behavior of the controlled variable of both output temperatures T_{ow} and T_{past} can be observed in Figures 5a and 5b, while the controlled variables track the references. The behavior of controlled temperatures, T_{ow} and T_{past}, from the pasteurization plant with one MPC controller are presented in Figure 5c together with their corresponding references. The output temperatures of hot water tank and heat exchanger in Figures 5c have the similar behaviors in comparison with the same variables in Figures 5a and 5b. As it can be seen in Figure 5c the settling times of hot water tank and heat exchanger are $\sim$ 500s and $\sim$ 120s, respectively. Besides the settling times of hot water tank and heat exchanger are $\sim$ 500s and $\sim$ 120s, in Figure 5a and 5b, respectively. Results obtained by applying first approach are depicted in Figure 6 and Figure 7. These figures shows a comparison between the measurement response of controlled variables

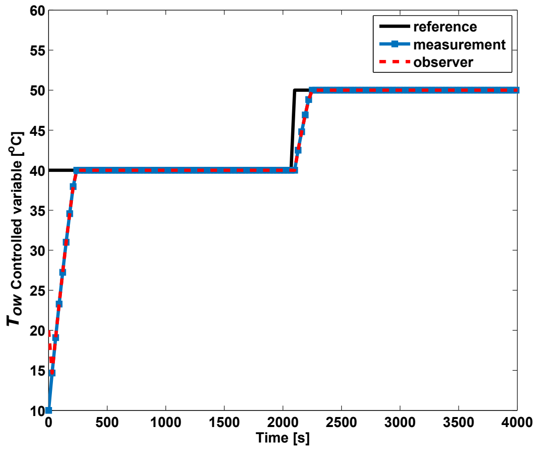


Figure 6. Controlled temperature (T_{ow}) in first Scenario.

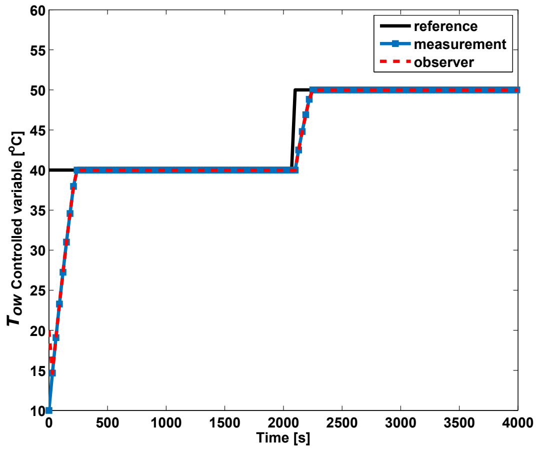


Figure 7. Controlled temperature T_{past} in first Scenario.

and observer output. Figure 6 refers to the temperature of hot water tank T_{ow}, where the complete system is separated into two subsystems. Figure 6 relate to the pasteurization temperature T_{past}, where one of the inputs is T_{ow}. In all cases, it can be seen that despite the difference in the initial conditions, the observer converges to the real state. The controlled and observed temperature responses from the pasteurization process using the second approach (that is, the inner state-feedback controller) are shown in Figure 8. The measurement outputs and observer outputs tracking the references also, the observer converges to measurement outputs after the effect of the initial conditions disappears.

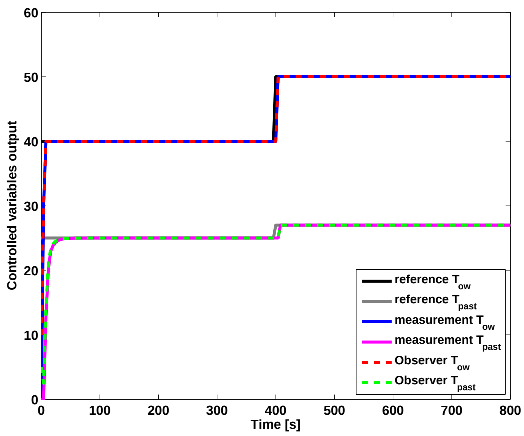


Figure 8. Controlled temperatures T_{ow} and T_{pas} with state-feedback controller (second Scenario).

5. CONCLUSIONS

In this paper, an output-feedback MPC controller has been designed for a pasteurization process, when an LPV mathematical model of the pasteurization plant obtained from experimental data is considered. However, the complete model of this process is not totally observable. Two different ways are proposed for solving this issue. First, the complete model is decomposed into two subsystems. Second, an inner state-feedback controller is used. Then, the output-feedback MPC controller based on the observer is implemented, while the control objective is tracked setpoint with different reference points.

According to the results discussion, the objectives of the pasteurization process are kept close to the reference points by using any of the both approaches proposed. This implies savings in energy because the accuracy and performance are achieved with output-feedback MPC controller and it is not necessary to create a setpoint with a safety differences above the accurate pasteurization temperature. As future research, the proposed approaches will be tested in the real pilot pasteurization plant.

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