Foundations for the Approximate Synthesis of RCCC Motion Generators

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Abstract The approximate synthesis of RCCC linkages for motion generation, a.k.a. rigid-body guidance, is the subject of this paper. A formulation is proposed here based on dual algebra, thereby leading to a *dual, constrained, nonlinear least-square problem*. The dual normality conditions necessary to obtain a feasible least-square approximation are established, following which an algorithm for the solution of the problem is proposed.

Key words: Spatial Burmester problem, approximate rigid-body guidance, dual normality conditions, dual, constrained, nonlinear least squares.

1 Introduction

The general problem of linkage synthesis consists in finding the dimensions of a linkage of a given topology—number of links, number of joints, types of joints, and number of kinematic loops—for a designated task. In this paper the task of interest is rigid-body guidance, as defined by Ludwig Burmester (1840-1927) [1] for the planar case, for which reason the problem is also named after Burmester. It is known that the planar and spherical Burmester problems allow for the synthesis of a four-bar linkage to guide their coupler link through up to five prescribed poses. For the spatial case, the four-bar linkage becomes of the RCCC type, where R stands for revolute, C for cylindrical joint. The linkage is usually synthesized via its two defining *dyads*, RC and CC; then, of the multiple solutions obtained for each dyad, one RCCC linkage is assembled upon coupling the dyads by means of the *coupler link*. Now, the number of parameters that determine a dyad as well as the number of constraints that each dyad type must satisfy are different for each of the two foregoing dyads. The maximum number of coupler poses that each dyad can visit

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exactly is five for the CC dyad, three for the RC (or CR) dyad [2]. Apparently, the maximum number of poses that a RCCC linkage can visit exactly is three, which is rather limited. However, if a condition is imposed that leads to a coupling of the two dyads, e.g., robustness to variations in the selection of the intermediate poses, as reported in [3], then a maximum of four poses can be visited with the RCCC linkage. The number of poses that can be met exactly is thus still limited, whence the motivation behind this paper.

In practice it is seldom required that intermediate poses be visited exactly. For example, if the linkage under design is to be used to deploy and retract an aircraft landing gear, only the deployed and the retracted poses of the wheel are to be met exactly, the intermediate ones being free to deviate from a prescribed trajectory, in pose space, as long as the deviations are within reasonable, prescribed limits and the various moving links do not collide with the fuselage or between themselves. It is thus apparent that the intermediate poses can be visited approximately, thereby allowing for *approximate synthesis*, the subject of the paper.

Approximate linkage synthesis is a classical subject, treated in some books [4], [5], [6], that has been approached as a problem of least squares.

2 Problem Formulation

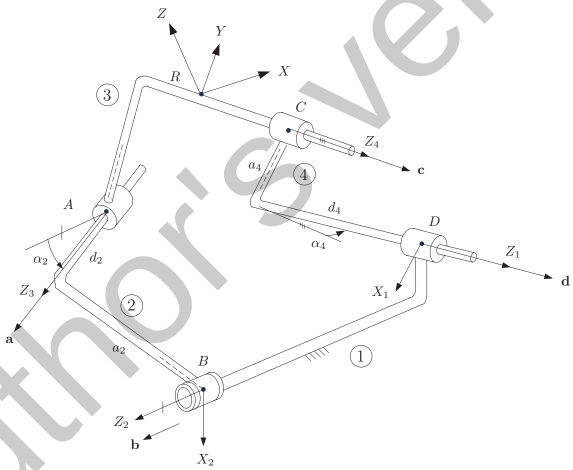

Fig. 1 A generic RCCC linkage

The linkage under synthesis bears the generic geometry depicted in Fig. 1, where link 1 is fixed, link 3 is the coupler, to which a frame $\mathscr{F}$ $\{X, Y, Z\}$ has been attached with origin at point R, while links 2 and 4 are coupled to link 1 by means of a R and a C joint, respectively, to link 3 by means of C joints. Moreover, links 2 and 4 are the RC and the CC dyads, respectively. Thus, axes Z_1 and Z_2 play the role of the center points, while axes Z_3 and Z_4 those of the circle points of the planar motion generator [7]. Apparently, axes Z_1 and Z_2 are grounded, and hence, remain stationary during the linkage motion, their counterparts Z_3 and Z_4 becoming lines of corresponding hyperboloids of revolution of axes Z_1 and Z_2, respectively. As a matter of fact, axes Z_3 and Z_4 become generators of the hyperboloids, which, in the general case, of arbitrary—not axially symmetric—single-sheet hyperboloids, are *reguli* of these surfaces. For these reasons, Z_1 and Z_2 will be termed the axis lines, or $\mathscr{A}$-lines, Z_3 and Z_4 the regulus lines, or $\mathscr{R}$-lines of the dyads under synthesis. For simplicity of notation, axes $Z_1, \ldots, Z_4$ will be denoted henceforth $\mathscr{B}$, $\mathscr{A}$, $\mathscr{C}$, $\mathscr{D}$, respectively. Moreover, $\mathscr{A}_0$ and $\mathscr{C}_0$ denote axes $\mathscr{A}$ and $\mathscr{C}$ at the reference pose of $\mathscr{F}$, with $\mathscr{A}_j$ and $\mathscr{C}_j$ denoting the location of $\mathscr{A}$ and $\mathscr{C}$ when $\mathscr{F}$ finds itself at its jth pose, for $j = 1, \ldots, m$.

Moreover, the motion under study is described by a set of poses $\mathscr{P} = \{\mathbf{r}_j, \mathbf{Q}_j\}_1^m$, where $\mathbf{r}_j$ is the position vector of R_j and $\mathbf{Q}_j$ is the orthogonal matrix that rotates frame $\mathscr{F}$ from its reference pose with origin at R_0 and orientation $\mathbf{Q}_0 \equiv \mathbf{1}$ to its jth pose. The purpose of linkage synthesis for motion generation in the case at hand consists in finding lines $\mathscr{A}_0$, $\mathscr{B}$, $\mathscr{C}_0$ and $\mathscr{D}$ that define completely the RCCC linkage, so that $\mathscr{F}$ will visit the set $\mathscr{P}$ with a *minimum error*. A word of caution is in order: vectors $\mathbf{r}_j$ having units of length and matrix $\mathbf{Q}_j$ being nondimensional, the error in missing a prescribed pose cannot be defined. In planar-linkage synthesis, the error is measured indirectly, in terms of the deviations of the various locations of the circle points from lying in a circle with center at one center point. In the same vein, the error in this case is measured as the distance of one $\mathscr{R}$-line from its corresponding axially symmetric hyperboloid. The error will be measured by mimicking exactly what is done in planar linkage synthesis: first find the circle that best fits a set of m center-point locations in the least-square sense; then, find the minimum distance of the putative center point in question to the circle, which is measured along the line that joins the putative circle point with the "center" point. In the spatial case under study, the distance from the $\mathscr{R}$-line to its corresponding $\mathscr{A}$-line consists of two items, the length of the segment of the common normal between $\mathscr{A}$ and $\mathscr{R}$ and the angle between the two lines, which can best be described by means of *dual algebra* [8]: Let $\mathbf{a}, \ldots, \mathbf{d}$ denote the unit vectors parallel to axes $\mathscr{A}, \ldots, \mathscr{D}$, respectively, the *moments*[1] of axes $\mathscr{A}, \ldots, \mathscr{D}$ being denoted by $\mathbf{a}_o, \ldots, \mathbf{d}_o$.

The problem can now be stated as: *Given the set $\mathscr{P}$ of $m > 4$ poses that the coupler link of a RCCC linkage is to visit, find lines $\mathscr{A}_0$, $\mathscr{B}$, $\mathscr{C}_0$ and $\mathscr{D}$ that define the RCCC linkage that carries its coupler link through the set $\mathscr{P}$ with a minimum error in the least-square sense.*

In order to formulate the problem, dual algebra [9], [10] is invoked. A dual unit vector $\hat{\mathbf{l}} = \mathbf{l} + \varepsilon \mathbf{l}_o$ represents a line $\mathscr{L}$ of direction given by the unit vector $\mathbf{l}$ and of moment $\mathbf{l}_o$ with respect to the origin. In this vein, the lines defining the RCCC link-

[1] The moment of a line in a given coordinate frame is defined as the cross product of the position vector of *any* point of the line times the unit vector parallel to the line. The mechanical interpretation of this concept is the moment of a unit force whose line of action is the line at stake.

age of Fig. 1 are represented by the dual unit vectors $\hat{\mathbf{a}}$, $\hat{\mathbf{b}}$, $\hat{\mathbf{c}}$ and $\hat{\mathbf{d}}$. Correspondingly, $\hat{\mathbf{a}}_0$ and $\hat{\mathbf{c}}_0$ represent lines $\mathscr{A}_0$ and $\mathscr{C}_0$, respectively, i.e., the reference locations of $\mathscr{A}$ and $\mathscr{C}$, with a similar notation for $\mathscr{A}_j$ and $\mathscr{C}_j$, for $j = 1, \ldots, m > 4$. Therefore,

$$\hat{\mathbf{a}}_j = \hat{\mathbf{Q}}_j \hat{\mathbf{a}}_0, \quad \hat{\mathbf{c}}_j = \hat{\mathbf{Q}}_j \hat{\mathbf{c}}_0, \quad j = 1, \ldots m \tag{1}$$

with $\hat{\mathbf{Q}}_j = \mathbf{Q}_j + \varepsilon \mathbf{Q}_{oj}$ denoting the dual orthogonal matrix that carries $\mathscr{F}$ from its reference pose to its jth pose. In this notation, $\mathbf{Q}_j$ denotes a rotation matrix, while $\mathbf{Q}_{oj} \equiv \mathbf{D}_j \mathbf{Q}_j$, and $\mathbf{D}_j$ denotes the cross-product matrix (CPM) of vector $\mathbf{d}_j$ that represents the translation of point R. The *cross product matrix* of a 3-dimensional vector $\mathbf{u}$ is defined as $\mathbf{U} = \mathrm{CPM}(\mathbf{u}) \equiv \partial(\mathbf{u} \times \mathbf{v})/\partial \mathbf{v}$, for any 3-dimensional vector $\mathbf{v}$.

The angle between two dual unit vectors $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$, representing lines $\mathscr{U}$ and $\mathscr{V}$, respectively, is denoted as $\hat{\theta}$. This angle occurs in the dot and the cross products of the two given vectors, in the form:

$$\hat{\mathbf{u}}^T \hat{\mathbf{v}} = \cos\hat{\theta}, \quad \hat{\mathbf{u}} \times \hat{\mathbf{v}} = \hat{\mathbf{w}} \sin\hat{\theta} \tag{2}$$

where $\hat{\mathbf{w}}$ is the dual unit vector normal to both $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$, i.e., a line that is normal to the two lines represented by $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$ and intersecting the two lines. Moreover,

$$\cos\hat{\theta} = \cos\theta - \varepsilon d \sin\theta, \quad \sin\hat{\theta} = \sin\theta + \varepsilon d \cos\theta \tag{3}$$

with d denoting the distance between $\mathscr{U}$ and $\mathscr{V}$. The dual angle $\hat{\theta}$ thus represents the *dual distance* between the two given lines. Representing the rigid-body condition that links 2 and 4 must obey at every prescribed pose is now straightforward: the dual distance between lines $\mathscr{A}_j$ and $\mathscr{B}$ as well as that between $\mathscr{C}_j$ and $\mathscr{D}$ must remain equal to that between their reference counterparts $\mathscr{A}_0$ and $\mathscr{B}$ and, correspondingly, $\mathscr{C}_0$ and $\mathscr{D}$, i.e., in light of eqs.(1),

$$\hat{\mathbf{b}}^T \hat{\mathbf{Q}}_j \hat{\mathbf{a}}_j = \hat{\mathbf{b}}^T \hat{\mathbf{a}}_0, \quad \hat{\mathbf{d}}^T \hat{\mathbf{Q}}_j \hat{\mathbf{c}}_j = \hat{\mathbf{d}}^T \hat{\mathbf{c}}_0, \quad j = 1, \ldots m$$

or, in homogeneous form,

$$\hat{\phi}_j \equiv \hat{\mathbf{b}}^T (\hat{\mathbf{Q}}_j - \mathbf{1}) \hat{\mathbf{a}}_0 = 0, \quad \hat{\phi}_{j+m} \equiv \hat{\mathbf{d}}^T (\hat{\mathbf{Q}}_j - \mathbf{1}) \hat{\mathbf{c}}_0 = 0, \quad j = 1, \ldots m \tag{4}$$

which represent $2m$ *dual synthesis equations* in the four dual unknown vectors $\hat{\mathbf{a}}_0$, $\hat{\mathbf{b}}$, $\hat{\mathbf{c}}_0$ and $\hat{\mathbf{d}}$. As each equation involves two real equations, one for its primal, one for its dual part, the total number of real equations is $4m$. Likewise, each dual unknown vector entails two 3-dimensional dual vectors, the total number of unknowns is 24. However, each dual unit vector must obey the unit-vector constraints:

$$\hat{h}_1 \equiv \|\hat{\mathbf{a}}_0\|^2 - 1 = 0,\ \hat{h}_2 \equiv \|\hat{\mathbf{b}}\|^2 - 1 = 0,\ \hat{h}_3 \equiv \|\hat{\mathbf{c}}_0\|^2 - 1 = 0,\ \hat{h}_4 \equiv \|\hat{\mathbf{d}}\|^2 - 1 = 0 \tag{5}$$

Again, each of the foregoing equations represents two real equations, one for its primal, one for its dual parts, thereby obtaining a total of eight real constraints. Moreover, the primal part refers to the normality of the primal unit vector, the dual part to the *Klein condition*[2]. The foregoing conditions apply to dyads of the CC type. The RC dyad must obey one more condition: *the sliding of the joint coupling link 2 with link 1 must vanish*, which can be enforced by stating that all common normals $\mathscr{N}_j$ to $\mathscr{A}_j$ and $\mathscr{B}$ must intersect $\mathscr{N}_0$, the counterpart normal to $\mathscr{A}_0$ and $\mathscr{B}$. Let $\hat{\mathbf{n}}_0$ represent $\mathscr{N}_0$, $\hat{\mathbf{n}}_j$ representing $\mathscr{N}_j$. The intersection condition can be expressed via the dual unit vectors representing the lines of interest. Indeed, from the expansion of $\cos\hat{\theta}$ in eq.(3), it is apparent that the intersection condition is that the dual part of $\hat{\mathbf{n}}_j^T\hat{\mathbf{n}}_0$, represented as $\mathrm{du}(\hat{\mathbf{n}}_j^T\hat{\mathbf{n}}_0)$, vanish, whence m additional constraints are obtained, namely,

$$h_{4+j} \equiv \mathrm{du}(\hat{\mathbf{n}}_j^T\hat{\mathbf{n}}_0) = 0, \quad j = 1, \ldots m \tag{6}$$

Further, the 12-dimensional dual vector of unknowns $\hat{\mathbf{x}}$ is introduced:

$$\hat{\mathbf{x}} = \left[\, \hat{\mathbf{a}}_0^T \; \hat{\mathbf{b}}^T \; \hat{\mathbf{c}}_0^T \; \hat{\mathbf{d}}^T \,\right]^T \tag{7}$$

together with the $2m$-dimensional dual vector $\hat{\phi}(\hat{\mathbf{x}})$ of synthesis equations, whose components are $\hat{\phi}_j$ and $\hat{\phi}_{j+m}$, as defined in eq.(4), and the $(4+m)$-dimensional dual vector of constraints $\hat{\mathbf{h}}(\hat{\mathbf{x}})$, whose components are defined in eqs.(5) and (6). Notice that, contrary to the exact synthesis case, here the synthesis equations need not be exactly satisfied; a reasonable approximation to those equations suffices. However, the $4+m$ constraints must be met exactly—up to roundoff error, of course. The optimization problem is now stated as one of *constrained nonlinear least squares*:

$$\hat{f}(\hat{\mathbf{x}}) \equiv \frac{1}{2}\hat{\phi}(\hat{\mathbf{x}})^T\mathbf{W}\hat{\phi}(\hat{\mathbf{x}}) \;\rightarrow\; \min_{\hat{\mathbf{x}}} \tag{8a}$$

subject to

$$\hat{\mathbf{h}}(\hat{\mathbf{x}}) = \mathbf{0}_{4+m} \tag{8b}$$

In the problem statement (8a), $\mathbf{W}$ is a symmetric, positive-definite weighting matrix that is introduced to allow for assigning different relevance to different poses. For example, the mth pose may be given much higher relevance than its intermediate counterparts. A better approach would be to raise the mth pose to the category of constraints, so that it would be met exactly. However, the total number of constraints should be smaller than 12, the number of unknowns; else, the problem would be overconstrained and no solution would be possible, i.e., $m < 8$.

3 The Dual Normality Conditions

The *first-order normality conditions* (FONC) for a *constrained nonlinear programming problem*, the class to which problem (8a & b) belongs, are well known in the

[2] This condition states that the primal and the dual parts of a dual unit vector must be mutually orthogonal.

case of problems defined by vectors over the real field [11]. In our case, all vectors are defined over the ring of dual numbers[3]. Paraphrasing those normality conditions, we have, for the case at hand:

$$\nabla \hat{f} + \hat{\mathbf{J}}^T \hat{\lambda} = \mathbf{0}_{2m} \tag{9}$$

where $\hat{\lambda}$ is the $(4+m)$-dimensional vector of dual Lagrange multipliers that are needed to take the constraints into account, and $\hat{\mathbf{J}}$ is the $2m \times (4+m)$ Jacobian matrix of the constraints, i.e., the gradient of $\hat{\mathbf{h}}$. Moreover, by virtue of the form of the objective function $\hat{f}$, $\nabla \hat{f}$ takes the form

$$\nabla \hat{f} = \hat{\boldsymbol{\Phi}}^T \mathbf{W} \hat{\boldsymbol{\phi}} \tag{10}$$

with $\hat{\boldsymbol{\Phi}}$ defined as $\nabla \hat{\boldsymbol{\phi}}$. Now, if the expression for the derivative of a dual function $\hat{f}(\hat{x})$ with respect to its dual argument is recalled, with $\hat{f}$ and $\hat{x}$ given by $\hat{f} = f + \varepsilon f_o$ and $\hat{x} = x + \varepsilon x_o$, namely [12],

$$\frac{\mathrm{d}\hat{f}}{\mathrm{d}\hat{x}} = \frac{\mathrm{d}f}{\mathrm{d}x} + \varepsilon \frac{\mathrm{d}f_o}{\mathrm{d}x} = \frac{\mathrm{d}\hat{f}}{\mathrm{d}x} \tag{11}$$

then $\hat{\boldsymbol{\Phi}}$ and $\hat{\mathbf{J}}$ become $\hat{\boldsymbol{\Phi}} = \partial \hat{\boldsymbol{\phi}} / \partial \mathbf{x}$ and $\hat{\mathbf{J}} = \nabla \hat{\mathbf{h}} = \partial \hat{\mathbf{h}} / \partial \mathbf{x}$, i.e., only the derivatives w.r.t. the primal part of the dual argument come into play in the foregoing gradients.

What condition (9) states is that, at a *stationary point* of problem (8a), $\nabla \hat{f}$ need not vanish, but must lie in the range of $\hat{\mathbf{J}}^T$, i.e., the overdetermined system (9) of $2m$ linear equations in the $4+m$ (< 12) unknowns, the number of dual Lagrange multipliers in $\hat{\lambda}$, must admit an exact solution. The FONC can be stated in two alternative forms:

$$[\mathbf{1} - \hat{\mathbf{J}}^T (\hat{\mathbf{J}}\hat{\mathbf{J}}^T)^{-1} \hat{\mathbf{J}}] \hat{\boldsymbol{\Phi}} \mathbf{W} \hat{\boldsymbol{\phi}} = \mathbf{0}_{4+m}, \quad \hat{\mathbf{L}}^T \hat{\boldsymbol{\Phi}}^T \mathbf{W} \hat{\boldsymbol{\phi}} = \mathbf{0}_{4+m} \tag{12}$$

with $\mathbf{0}_{4+m}$ denoting the $(4+m)$-dimensional zero vector.

The matrix inside the brackets in the first of the foregoing equations can be readily identified as a projector that maps n-dimensional vectors onto the null space of $\hat{\mathbf{J}}$. Matrix $\hat{\mathbf{L}}$ in the second equation is a $12 \times (n-4-m)$ orthogonal complement of $\hat{\mathbf{J}}$, i.e, $\hat{\mathbf{J}}\hat{\mathbf{L}} = \mathbf{O}$, with $\mathbf{O}$ denoting the $(4+m) \times (n-4-m)$ zero matrix.

These conditions are necessary for a value of $\hat{\mathbf{x}}$ to be a *stationary point* of the problem under study. For this point to be a minimum, the *second-order normality condition* must be satisfied. In nonlinear-programming problems, this condition is that the *reduced Hessian* of the problem under study be positive-definite at a stationary feasible point, i.e., at a point that satisfies both the FONC, eqs.(12), and the constraints, eqs.(8b). In our case, such a point, designated by $\hat{\mathbf{x}}^*$, is assumed to have been found, vector $\hat{\boldsymbol{\phi}}(\hat{\mathbf{x}}^*)$ being represented by $\hat{\boldsymbol{\phi}}^*$. The reduced Hessian matrix takes the form

[3] While vector spaces must be defined over a field, in our context we need to define them over the set of dual numbers, that do not form a field, but rather a ring. This difference does not pose any technical problem to the developments in the balance of the paper.

$$\hat{\mathbf{H}}_r = \hat{\mathbf{L}}^T \left[\hat{\Phi}^T \mathbf{W} \hat{\Phi} + \frac{\partial(\hat{\Phi}^T \mathbf{W} \hat{\phi}^*)}{\partial \hat{\mathbf{x}}} + \frac{\partial(\hat{\mathbf{J}}^T \hat{\lambda})}{\partial \hat{\mathbf{x}}} \right] \hat{\mathbf{L}} \tag{13}$$

In our case, the second-order normality condition for a minimum is that the primal part of $\hat{\mathbf{H}}_r$ be positive-definite.

4 The Dual Orthogonal-decomposition Algorithm

The orthogonal-decomposition algorithm (ODA) was developed by the author and his team to solve equality-constrained problems in mathematical programming [13]. When applied to the RCCC approximate-synthesis problem, the algorithm takes the form described below: it is assumed that a feasible approximation to the optimum has been obtained at the kth iteration, $\hat{\mathbf{x}}^k$, an increment $\Delta\hat{\mathbf{x}}^k$ being sought that will yield an improved approximation $\hat{\mathbf{x}}^{k+1}$. The strategy consists in decomposing the increment in two parts, namely,

$$\Delta\hat{\mathbf{x}}^k = \Delta\hat{\mathbf{v}}^k + \hat{\mathbf{L}}_k \Delta\hat{\mathbf{u}}^k \tag{14}$$

with $\hat{\mathbf{L}}_k$ denoting the orthogonal complement $\hat{\mathbf{L}}$ evaluated at $\hat{\mathbf{x}}^k$. Moreover, $\Delta\hat{\mathbf{v}}^k$ is the minimum-norm solution of the underdetermined linear system of dual equations

$$\hat{\mathbf{J}}_k \Delta\hat{\mathbf{v}}^k = -\hat{\mathbf{h}}^k \tag{15}$$

in which $\hat{\mathbf{J}}_k$ and $\hat{\mathbf{h}}^k$ denote the Jacobian $\hat{\mathbf{J}}$ and vector $\hat{\mathbf{h}}$ evaluated at $\hat{\mathbf{x}}^k$. The minimum-norm solution of eq.(15) can be expressed in terms of the dual right Moore-Penrose generalized inverse [14], namely,

$$\Delta\hat{\mathbf{v}}^k = -\hat{\mathbf{J}}_k^\dagger \hat{\mathbf{h}}^k, \quad \hat{\mathbf{J}}_k^\dagger \equiv \hat{\mathbf{J}}_k^T (\hat{\mathbf{J}}_k \hat{\mathbf{J}}_k^T)^{-1} \tag{16}$$

where $\hat{\mathbf{J}}_k^\dagger$ is to be calculated with the dual QR-decomposition of $\hat{\mathbf{J}}_k^T$. The QR-decomposition for real matrices is well documented in the literature on numerical analysis [15]. With $\Delta\hat{\mathbf{v}}^k$ computed, $\Delta\hat{\mathbf{u}}^k$ is computed as the least-square approximation of an overdetermined system of linear equations:

$$\mathbf{V}\hat{\Phi}_k \hat{\mathbf{L}}_k \Delta\hat{\mathbf{u}}^k = -\mathbf{V}(\hat{\phi}^k + \hat{\Phi}_k \Delta\hat{\mathbf{V}}^k), \quad \mathbf{W} \equiv \mathbf{V}^T \mathbf{V} \tag{17}$$

whence the solution $\Delta\hat{\mathbf{u}}^k$ is computed with the left Moore-Penrose generalized inverse of the product $\mathbf{V}\hat{\Phi}_k\hat{\mathbf{L}}_k$:

$$\Delta\hat{\mathbf{u}}^k = -(\mathbf{V}\hat{\Phi}_k \hat{\mathbf{L}}_k)^I (\hat{\phi}^k + \hat{\Phi}_k \Delta\hat{\mathbf{v}}^k), \quad (\mathbf{V}\hat{\Phi}_k \hat{\mathbf{L}}_k)^I \equiv (\hat{\mathbf{L}}^T \hat{\Phi}^T \mathbf{W} \hat{\Phi} \hat{\mathbf{L}})^{-1} \hat{\mathbf{L}}^T \hat{\Phi}^T \mathbf{V} \tag{18}$$

thereby completing the $(k+1)$st iteration. The procedure stops when the FONC, eqs.(12), are verified to a prescribed tolerance.

5 Conclusions

The foundations for the approximate synthesis of RCCC linkages for motion generation were laid down. It was shown that, by virtue of the normality conditions that the dual unit vectors that represent the linkage four joint axes must observe, the number of prescribed poses of the coupler link is limited to being smaller than eight.

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