Kinematic Design of Two Elementary 3DOF Parallel Manipulators with Configurable Platforms

Antonius G.L. Hoevenaars, Patrice Lambert and Just L. Herder

Abstract Parallel Manipulators with Configurable Platforms (PMCPs) have platforms with internal degrees of freedom and form a class of manipulators that is not covered by existing type synthesis methods. Because the minimum number of legs for a PMCP is three, fully parallel 3DOF PMCPs may be considered an elementary subset of PMCPs. To support the extension of type synthesis methods to PMCPs, this paper presents the first kinematic designs of manipulators from this subset. A structured design method has led to the kinematic design of two spatial manipulators that are both capable of independently performing one translation, one rotation and one internal platform motion.

Key words: parallel manipulator, configurable platform, 3DOF, grasping motion, spatial.

1 Introduction

Robotic manipulation sometimes requires additional degrees of freedom (DOF) such as grasping on top of the rigid end-effector motion. Multiple solutions have been proposed to achieve this additional motion. One example is to combine two separate mechanisms [4] and another is to attach a gripper mechanism in series to the end-effector, as is the case in the commercial omega.7 by Force Dimension. The

Antonius G.L. Hoevenaars
Department of Precision and Microsystems Engineering, Delft University of Technology
Mekelweg 2, 2628 CD Delft, the Netherlands e-mail: A.G.L.Hoevenaars@TUDelft.nl
Patrice Lambert
Department of Precision and Microsystems Engineering, Delft University of Technology
Mekelweg 2, 2628 CD Delft, the Netherlands e-mail: P.Lambert@TUDelft.nl
Just L. Herder
Department of Precision and Microsystems Engineering, Delft University of Technology
Mekelweg 2, 2628 CD Delft, the Netherlands e-mail: J.L.Herder@TUDelft.nl

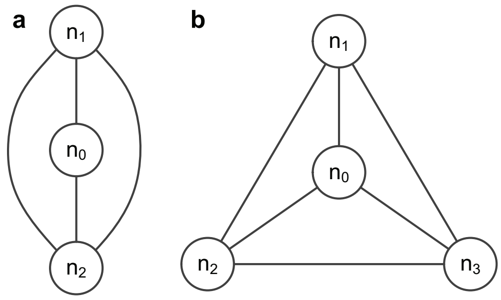

Fig. 1 **a** Using graph theory a PMCP mechanism with two legs can be represented as a series-parallel mechanism with a base n_0 and two link nodes n_1 and n_2, **b** A PMCP with three legs cannot be represented as a series-parallel mechanism, but yields a so-called wheel graph

first solution increases the complexity of the system while the latter adds the inertia of an additional motor to the end-effector. Because low inertia at the end-effector is one of the distinguishing features of parallel manipulators, additional inertia especially affects the performance of parallel manipulators.

In the past decade it has been recognised that additional DOF can also be added to the end-effector of a parallel manipulator without compromising its parallel structure. This is achieved by replacing the rigid end-effector with an additional closed loop. Following the 4DOF planar manipulator with grasping motion by Yi et al. [10], Mohamed and Gosselin generalised the analysis of this new class of manipulators called Parallel Manipulators with Configurable Platforms (PMCP) [8]. Other examples of such PMCPs are the Par4 by Nabat et al. [9] and a 5DOF design by Lambert et al. [6].

An illustrative method to discuss kinematic structures is graph theory [1], which represents every mechanism as a series of joints (lines) and rigid bodies (nodes). Fig. 1 illustrates how in graph theory a PMCP with two legs is kinematically equivalent to a *series-parallel architecture*, while a PMCP with three legs is not; in fact it belongs to a different category labelled *non-series-parallel architectures* [5]. PMCPs with three legs (serial chains) may therefore be regarderd as the most basic subset of PMCP designs. In this paper only *fully parallel manipulators* are considered, for which the number of legs is strictly equal to the number of DOF of the end-effector [7]. Thus, if only the joints located at the base are actuated, three legs allow 3DOF. Consequently, it is argued in this paper that fully parallel 3DOF PMCPs represent an elementary subset of PMCP designs.

Interestingly, PMCPs discussed in the literature all have a minimum of 4DOF. They have not been developed using a type synthesis method such as the one introduced by Kong and Gosselin [3] or Gogu [2], since existing methods do not cover PMCPs. Because fully parallel 3DOF PMCPs are argued to form an elementary subset of PMCP designs, examples from this subset may provide interesting input for the future development of a type synthesis method that does cover PMCPs.

The goal of this paper is to verify the existence of fully parallel 3DOF PMCPs and present the first architectures from this elementary subset. The structure of the paper is as follows. First the design method is discussed that leads to the two kinematic architectures presented in this paper. Next, the inverse Jacobian is derived for one of the two kinematic designs and four singular configurations are identified.

2 Design Method

Because no type synthesis method exists for PMCPs, the method in this paper relies on the structured combination of a four-bar mechanism (the platform) with a set of pre-defined legs. Furthermore, two restricting conditions are posed on the designs. The first condition is that the resulting 3DOF PMCPs shall be fully parallel. The number of legs is therefore strictly limited to three. Secondly, the axis associated with each DOF shall coincide with an axis of either the inertial reference frame XYZ or the platform reference frame $X^*Y^*Z^*$. This condition facilitates a straightforward description of the resulting mobilities.

The design method applied in this paper consists of four steps. First, the building blocks are defined: a planar four-bar mechanism and three identical legs. A four-bar mechanism with links of equal length is used, which is known to have three overconstraints and one internal DOF. Thus, the total number of platform DOF is seven. The internal DOF is expressed as the distance P_g between one of the joints and the platform reference frame origin. On the premise that a fully parallel 3DOF PMCP requires each of the three legs to have a minimum of three DOF, a minimal leg consists of two links and three joints and describes planar motion. One of the end joints is connected to an actuator at the base. In this paper the choice was made to use rotating actuators but this choice does not impact the DOF of the individual legs. The described building blocks are shown in Fig. 2 **a**.

The second step is to constrain the motion of the platform reference frame origin to a plane, which is achieved through the connection of two legs to opposite joints of the four-bar mechanism. These legs are connected such that the resulting plane of motion of the platform reference frame origin is perpendicular to either X^* or Y^*. This is to ensure that the remaining DOF are all alligned with an axis of either the inertial reference frame or the platform reference frame. The plane of motion of the platform reference frame origin is here defined as the XZ-plane, as shown in Fig. 2 **b**. The mechanism now has four DOF.

In the third step an additional DOF is constrained using the third leg. To constrain the platform in another DOF, the third leg is oriented in either of the planes perpendicular to the first two legs. Connecting the third leg in this orientation to one of the two remaining platform joints adds two additional constraints (and one overconstraint) to the platform. The state of the two kinematic designs after this step is shown in Figs. 2 **c** and **d**.

By constraining five of the original seven DOF, both mechanisms shown in Figs. 2 **c** and **d** have two DOF remaining. The final step is therefore to relieve one of the constrained DOF by introducing an additional joint. For the mechanism shown in Fig. 2 **d** this also requires a change in the orientation of the joint connecting the third leg to the platform. The two resulting kinematic designs are shown in Figs 2 **e** and **f**.

This section has described the kinematic design of two fully parallel 3DOF PMCPs, both of which have four overconstraints. In graph theory notation, both architectures are represented by the graph in Fig. 3 which is equivalent to the one shown in Fig. 1 **b** after serial reductions [5].

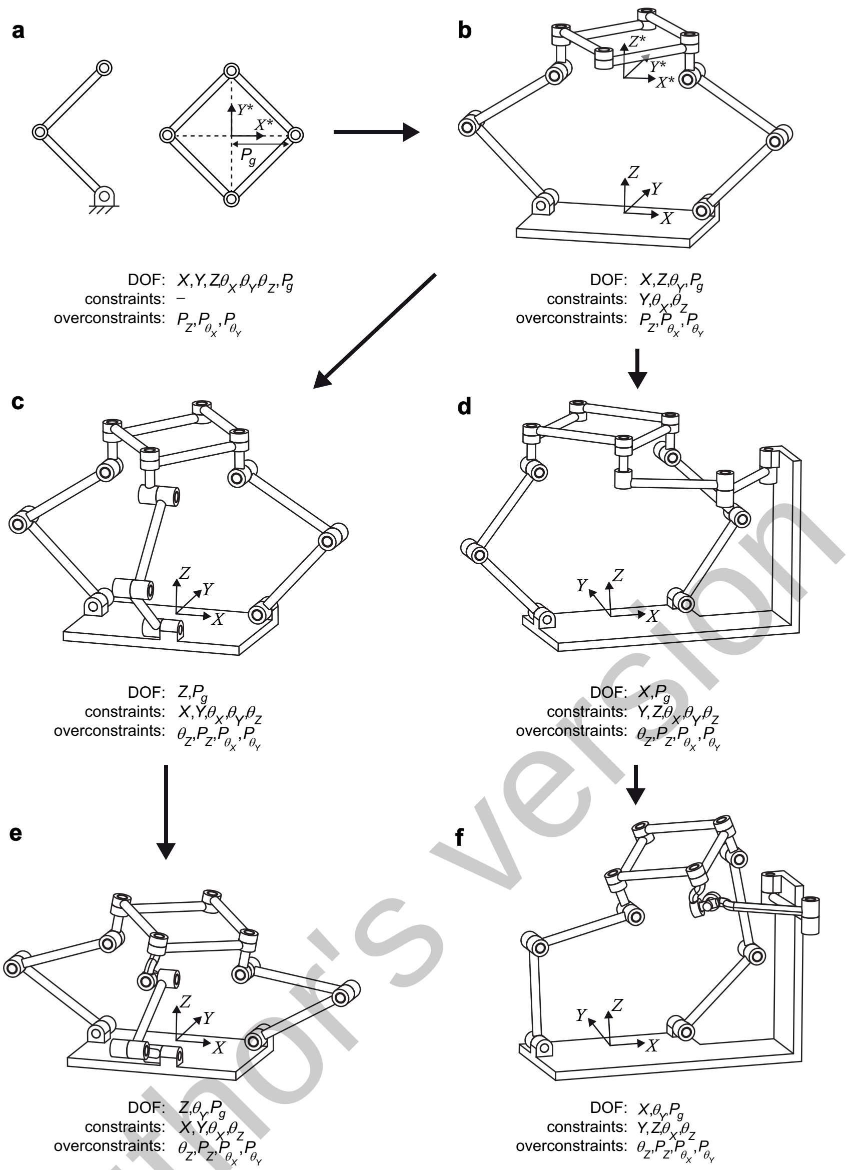

Fig. 2 **a** the minimum building blocks for a fully parallel 3DOF PMCP, **b** the mechanism after connection of the first two legs, **c**, **d** the two possibilities for connecting the third leg, **e**, **f** the two resulting fully parallel 3DOF PMCPs

3 Derivation of Inverse Jacobian

In this section the inverse Jacobian is derived for the manipulator introduced in Fig. 2 **e**, which is also shown in Fig. 4 including the notations that are used in this

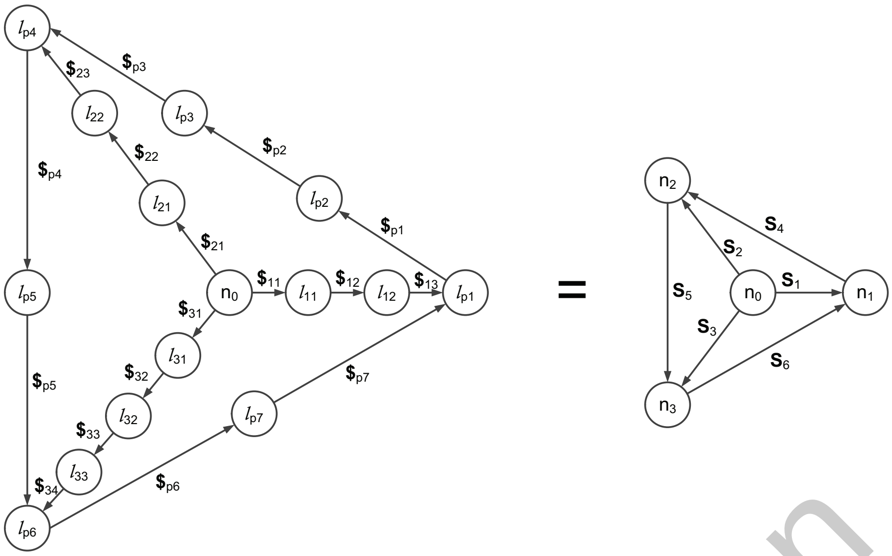

Fig. 3 Representation of the mechanisms shown in Figs. 2 **e** and **f** using graph theory where l_{il} stands for the l^{th} link of the leg i and $\$_{ij}$ for the screw associated with joint j of leg i, while in the reduced graph after serial reduction n_i and $\mathbf{S}_i$ are respectively the i^{th} link node and screw system

section. For the purpose of easy analysis, the end-effector forces acting in the direction of the manipulator DOF are here expressed as forces acting on two specific end-effector points (see Fig. 4). However, in practice the complete configurable platform may act as end-effector, for example when grasping a deformable object.

Before the inverse Jacobian is derived, it is first confirmed that the mobility M and overconstraints R_C presented in Fig. 2 **e** are consistent with the Chebychev–Grübler–Kutzbach criterion. It was observed that both resulting designs have four overconstraints, $R_C = 4$. The links and joints can be counted easily using the graph theory representation in Fig. 3, which counts $n = 15$ links l_{ij} (of which link l_{33} has zero length) and $m = 17$ joints with an associated screw $\$_{ij}$. All joints have one DOF, so $f_m = 1$ for all m joints. Because in the original Chebychev–Grübler–Kutzbach criterion any overconstraints are included in the resulting mobility, the criterion is often rewritten to

$$M = 6(n-m-1) + \sum_{i=1}^{m} (f_m) + R_C \tag{1}$$

which equals $M = 3$ if the above numbers are used. This is consistent with the expected mobility. The remainder of this section deals with the inverse Jacobian derivation for the mechanism shown in Fig. 4. More precisely, the transpose of the inverse Jacobian (J^{-T}) is derived, mapping the actuator torques on the end-effector forces according to

$$\mathbf{F}^* = J^{-T}\overline{\tau} \tag{2}$$

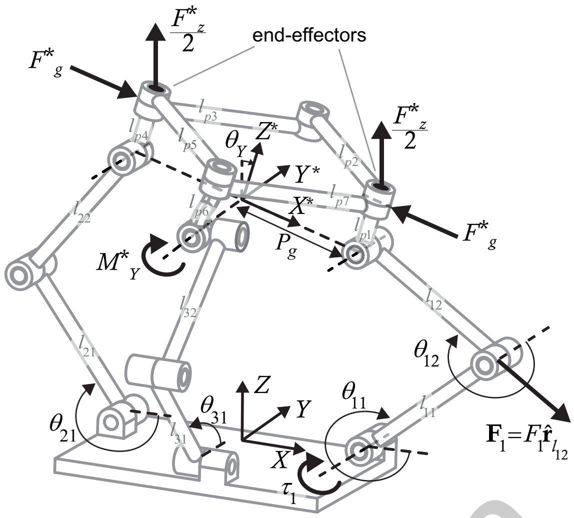

Fig. 4 3DOF PMCP that can independently translate in Z, rotate around Y^* and perform grasping by means of the local platform DOF P_g

Although linear motors can also be used, the situation is here considered for rotary actuators located at the base. The torques τ_i applied by these actuators are transferred to the platform via forces $\mathbf{F}_i$ directed along links l_{i2}. Because all three legs are equal, $l_{11} = l_{21} = l_{31} = l_1$ and $\mathbf{F}_i$ can expressed using

$$\mathbf{F}_i = F_i \widehat{\mathbf{r}}_{l_{i2}} = \frac{\tau_i}{l_1 \sin(\theta_{i2})} \widehat{\mathbf{r}}_{l_{i2}} \tag{3}$$

These forces are transferred to the platform and can be expressed in terms of forces acting in the direction of the platform DOF. In this paper the internal platform DOF is considered a grasping motion with variable P_g [m], which is acted on by a force F^*_g. The other forces are M^*_Y around Y^* and F^*_Z in the Z-direction as indicated in Fig 4. With $l_{p2} = l_{p3} = l_{p5} = l_{p7} = l_p$, the expression of these forces in terms of forces $\mathbf{F}_i$ is

$$\begin{aligned}
\begin{bmatrix} F^*{}_g \\ F^*{}_Z \\ M^*{}_Y \end{bmatrix} &= \begin{bmatrix} -\cos(\theta_Y) & 0 & \sin(\theta_Y) \\ 0 & 0 & 1 \\ -P_g \sin(\theta_Y) & 0 & -P_g \cos(\theta_Y) \end{bmatrix} \mathbf{F}_1 \\
&+ \begin{bmatrix} \cos(\theta_Y) & 0 & -\sin(\theta_Y) \\ 0 & 0 & 1 \\ P_g \sin(\theta_Y) & 0 & P_g \cos(\theta_Y) \end{bmatrix} \mathbf{F}_2 \\
&+ \begin{bmatrix} 0 & -P_g \sqrt{\left({l_p}^2 - {P_g}^2\right)} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{F}_3
\end{aligned} \tag{4}$$

By combining Eq. 3 and Eq. 4 the platform forces can be directly expressed as a function of the actuator torques. For the manipulator shown in Fig. 4 this results in

$$\begin{bmatrix} F^*{}_g \\ F^*{}_Z \\ M^*{}_Y \end{bmatrix} = \frac{1}{l_1} \begin{bmatrix} \frac{-\cos(\theta_Y-\theta_{11}-\theta_{12})}{\sin(\theta_{12})} & \frac{\cos(\theta_Y-\theta_{21}-\theta_{22})}{\sin(\theta_{22})} & \frac{-P_g\cos(\theta_{31}+\theta_{32})}{\sin(\theta_{32})\sqrt{(l_p{}^2-P_g{}^2)}} \\ \frac{-\sin(\theta_{11}+\theta_{12})}{\sin(\theta_{12})} & \frac{-\sin(\theta_{21}+\theta_{22})}{\sin(\theta_{22})} & \frac{\sin(\theta_{31}+\theta_{32})}{\sin(\theta_{32})} \\ \frac{-P_g\sin(\theta_Y-\theta_{11}-\theta_{12})}{\sin(\theta_{12})} & \frac{P_g\sin(\theta_Y-\theta_{21}-\theta_{22})}{\sin(\theta_{22})} & 0 \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} \quad (5)$$

which is the expression of Eq. 2 for the manipulator shown in Fig. 4. Because of the power conservation principle the matrix J^{-T} in Eq. 5 can also be used in the velocity relation $\dot{\mathbf{q}} = J^{-1}\dot{\mathbf{x}}$ between the actuator velocities $\dot{\mathbf{q}}$ and the platform velocities $\dot{\mathbf{x}}$. Finally, the matrix J^{-T} can be analysed to reveal some of the characteristics of the developed manipulator, because in singular configurations the rank of J^{-T} reduces. For the manipulator presented in Fig. 4 singularities occur if

- the distance between the end-effectors is zero, $P_g = 0$
- the distance between the end-effectors is maximal, $P_g = l_p$
- one of the legs is completely extended or folded, $\theta_{i2} = \{0, \pi, ..\}$
- for both leg one and leg two, link l_{i2} is in line with the platform, $\theta_Y - \theta_{i1} - \theta_{i2} = \{0, \pi, ..\}$ for $i = \{1, 2\}$

4 Conclusion

This paper has presented the first kinematic designs of fully parallel 3DOF PMCPs, which were identified as an elementary subset of PMCP designs. The resulting mechanisms are spatial manipulators that can be independently controlled in one rotation, one translation and one internal platform motion. For one of the introduced 3DOF PMCPs the inverse Jacobian was derived, which has also allowed the identification of four singular configurations. Since existing type synthesis methods do not cover PMCPs, this paper has applied a structured, but not yet formalised, design method. Because the presented manipulators are considered to be part of an elementary subset of PMCP designs, they may prove to be useful input for the development of a type synthesis method that does cover PMCPs.

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