Four-Dimensional Persistent Screw Systems of the General Type

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Abstract When a mechanism moves, the twist system S of the end-effector generally varies. In significant special cases, S is a subalgebra of the Lie algebra of the special Euclidean group, and it remains constant. In more general cases, S remains invariant up to a proper isometry, thus preserving its *class*. A mechanism of this kind is said to generate a *persistent screw system* (PSS) of the end-effector. PSSs play an important role in mobility analysis and mechanism design. This paper presents the serial generators of 4-dimensional PSSs with a constant class of the *general* type.

Key words: Screw theory, mechanism synthesis, mobility analysis.

1 Introduction

Screw systems are the subspaces of the Lie algebra $se(3)$ of the Euclidean group $SE(3)$. Two screw systems are equivalent if one may be moved onto the other by a rigid-body displacement [4]. This equivalence relation divides the space of screw systems into infinitely many *classes*. The latter may be grouped into a finite number of general or special *types* [7, ch. 12], within which the constituent classes are identified by the values of a small number of parameters. Screw systems belonging to the same class have the same *dimension*, *type* and *shape*, thus differing only in their *pose* in space.

For a mechanism in a configuration Θ, the possible instantaneous motions of the end-effector are given by a screw system $S(\Theta) \subset se(3)$. In general, when Θ changes, so does $S(\Theta)$. An important special case occurs when the mechanism constrains the body to trace out (at least locally) a *subgroup* of $SE(3)$. Then, for any nonsingular Θ, $S(\Theta) = A$, where A is the *algebra* of the subgroup. The mechanism is said to gen-

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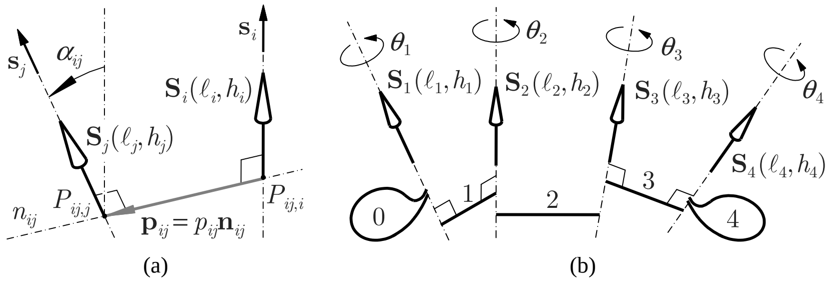

Fig. 1 Relative pose between two screws (a); a chain of four 1-dof lower pairs (b).

erate an *invariant screw system* (ISS) of the end-effector [7, 4, 5, 6]. Herein, a more general case is considered, namely a mechanism where $S(\Theta)$, although not necessarily constant, has a *constant class* . In other terms, $S(\Theta)$ retains its shape while it changes its pose, in effect moving like a rigid body in space. If that is so, and $\dim S(\Theta) = n$, the mechanism is said to generate an n-dimensional *persistent screw system* of the end-effector, or briefly an n-PSS. PSSs were presented by Carricato and Rico Martínez, who showed that PSS generators may be obtained by serially composing generators of ISSs [2, 3, 1]. The exhaustive classification of PSS generators is in progress. This paper complements the results in [1], where the generators of 4-PSSs with constant classes of *special* types were described. Here, the serial generators of 4-PSSs with constant classes of the *general* type are presented.

In the following, the locutions '*nG* system' and '*nR* system', with R being a Roman numeral, denote n-dimensional screw systems, respectively, of the *general* type and of the Rth *special* type, according to [7]. A normalized screw representing a relative twist between two bodies is designated by $\mathbf{S}$. The axis (when it exists), the pitch and the direction of $\mathbf{S}$ are denoted by ℓ, h and $\mathbf{s}$, respectively. When it is useful, ℓ and h accompany $\mathbf{S}$ within parentheses, i.e. $\mathbf{S}(\ell,h)$ (if $h=\infty$, ℓ is replaced by $\mathbf{s}$). Given two screws $\mathbf{S}_i(\ell_i,h_i)$ and $\mathbf{S}_j(\ell_j,h_j)$ (Fig. 1a), n_{ij} is the common normal between ℓ_i and ℓ_j; $P_{ij,i}$ and $P_{ij,j}$ are the feet of n_{ij} on ℓ_i and ℓ_j; $\mathbf{n}_{ij}$ is a unit vector parallel to n_{ij} and directed from $P_{ij,i}$ to $P_{ij,j}$; p_{ij} and α_{ij} are the shortest distance and the relative angle between ℓ_i and ℓ_j, with α_{ij} being measured about $\mathbf{n}_{ij}$ in the interval $(-\pi/2,\pi/2]$; finally, $\mathbf{p}_{ij} = P_{ij,j} - P_{ij,i} = p_{ij}\mathbf{n}_{ij}$.

2 Generators of 4-dimensional PSSs of the general type

Any subgroup of $SE(3)$ may be generated (at least locally) by a serial chain composed by 1-dof lower pairs. Hence, any 4-PSS generator emerging by the serial composition of ISSs may be considered 'equivalent' to a serial linkage $\mathfrak{S}$ composed by (at least) four 1-dof joints (Fig. 1b). Since joint motions affect neither the pitches nor the relative pose of adjacent joints, the geometry of the chain is completely defined by the joint screws at an arbitrarily-chosen nonsingular reference configuration, i.e. $\mathbf{S}_i = \mathbf{S}_i(\Theta)|_{\Theta=\mathbf{0}} = \mathbf{S}_i(\mathbf{0})$, $i = 1\ldots4$. Accordingly, $\mathfrak{S}$ may be identified with the array $\langle \mathbf{S}_1,\ldots,\mathbf{S}_4\rangle$. Link 0 of $\mathfrak{S}$ is the predecessor of $\mathbf{S}_1$; link i, with $i = 1\ldots3$, is the body laid between $\mathbf{S}_i$ and $\mathbf{S}_{i+1}$; and link 4 is the successor of $\mathbf{S}_4$.

After a displacement $\Theta = (\theta_1,\ldots,\theta_4)$, the ith joint screw, $i > 1$, is moved to $\mathbf{S}_i(\Theta) = \prod_{j=1}^{i-1}\mathbf{D}_j(\theta_j)\mathbf{S}_i$, where $\mathbf{D}_j$ is the adjoint action of the jth joint displacement. Generally, in the new configuration $S(\Theta) = \text{span}[\mathbf{S}_1(\Theta),\ldots,\mathbf{S}_4(\Theta)] \neq S(\mathbf{0})$. $\mathfrak{S}$ generates a PSS if, for every nonsingular Θ, $S(\Theta) = \mathbf{G}(\Theta)S(\mathbf{0})$, where $\mathbf{G}(\Theta)$ is the adjoint action of a *proper isometry*. A simpler formulation of this condition emerges by observing that joint motions θ_1 and θ_4 cannot alter the shape of $S(\Theta)$, since they do not affect the relative pose of $\mathfrak{S}$'s joint screws. Accordingly, when studying the persistent properties of $S(\Theta)$, θ_1 and θ_4 may be kept constant, e.g. $\theta_1 = \theta_4 = 0$, and link 2 may be conveniently chosen as the reference frame [2]. As a consequence, $\mathfrak{S}$ generates a 4-PSS if and only if, for every nonsingular pair (θ_2,θ_3), there is an adjoint action $\mathbf{G}(\theta_2,\theta_3)$ such that $S(\theta_2,\theta_3) = \mathbf{G}(\theta_2,\theta_3)S(0,0)$, where $S(\theta_2,\theta_3) = \text{span}\left[\mathbf{D}_2^{-1}(\theta_2)\mathbf{S}_1,\mathbf{S}_2,\mathbf{S}_3,\mathbf{D}_3(\theta_3)\mathbf{S}_4\right]$.

$S(\Theta)$ has a constant class of the *general* type if the principal screws $\mathbf{S}_{r1}(\ell_{p1},-h_{p1})$ and $\mathbf{S}_{r2}(\ell_{p2},-h_{p2})$ of the cylindroid $S^{\perp}$ reciprocal to S have constant *finite* pitches, and $h_{p1} \neq h_{p2}$ [7] (Fig. 2). If that is so, the principal screw $\mathbf{S}_{pi}(\ell_{pi},h_{pi})$ of S, with $i = 1,2$, is collinear with $\mathbf{S}_{ri}$ and has pitch h_{pi}, whereas the principal screws $\mathbf{S}_{p3}$ and $\mathbf{S}_{p4}$ span a cylindrical ISS $\mathscr{C}(\ell_{p3})$ along the nodal line of $S^{\perp}$ (the latter is the line perpendicular to ℓ_{p1} and ℓ_{p2}, passing through their intersection point O). $\mathscr{C}(\ell_{p3})$ and the ∞-pitch screw therein are, respectively, the only available ISS with dimension greater than 1 and the only ∞-pitch screw in S.

A generator $\mathfrak{S} = \langle \mathbf{S}_1,\ldots,\mathbf{S}_4\rangle$ of a 4G-PSS may be constructed by composing the ISSs available in S. Since $\mathscr{C}(\ell_{p3})$ has dimension 2 and S has dimension 4, no less than three ISSs need to be composed. A *ternary* generator is constructed by composing $\mathscr{C}(\ell_{p3})$ with two 1-dimensional ISSs, i.e. with two single screws. A *quaternary* generator emerges by composing four distinct screws in S. Since the 4G system comprises a single ∞-pitch screw, $\mathfrak{S}$ cannot include: more than one ∞-pitch screw; more than a pair of adjacent parallel finite-pitch screws; a pair of adjacent parallel finite-pitch screws plus an ∞-pitch screw. Otherwise, configurations in which ∞-pitch screws along more than one direction would appear.

$\mathfrak{S}$ may be synthesized by expanding the 3-PSS generators disclosed in [3]. Since the only 3-systems contained in a 4G system are the 3G, 3I, 3III, 3VII and 3$VIII$ systems, the only 3-PSSs that may appear within a 4G system are the 3I- and the 3$VIII$-PSS (cf. [3]). For this reason, these two will be the 'building blocks' of the generators described hereafter.

2.1 The ternary generator

Let $\mathfrak{B} = \langle \mathbf{S}_2(\ell_2,h_2),\mathbf{S}_3(\ell_3,h_3),\mathbf{S}_4(\ell_4,h_4)\rangle$ and let $\mathfrak{B}$ form a 3I-PSS B. For the properties of the 3I systems, all screws of B have finite pitch and, if $\mathbf{S}_{pi}^{B}(\ell_{pi}^{B},h_{pi}^{B})$,

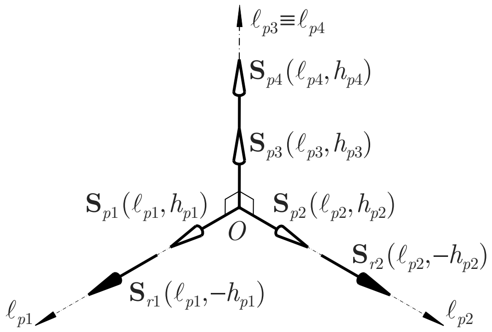

Fig. 2 Principal screws of a 4-system of the general type: $h_{p1} \neq \infty \neq h_{p2}$, and $h_{p1} \neq h_{p2}$.

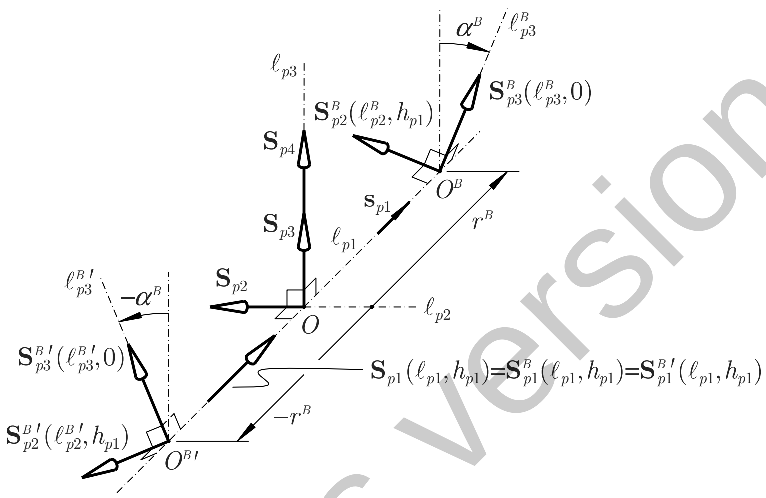

Fig. 3 $3I$ systems within a $4G$ system.

$i = 1 \ldots 3$, is the ith principal screw of B, then $h^B_{p1} = h^B_{p2}$ [7]. Since B is persistent, the following conditions also apply [3]:

$$h_3 = h^B_{p3} = 0, \quad \mathbf{S}_3(\ell_3, 0) = \mathbf{S}^B_{p3}(\ell^B_{p3}, 0), \quad h^B_{p1} = h^B_{p2} \neq 0, \quad P_{32,3} \equiv P_{34,3} \equiv O^B, \tag{1}$$

where O^B is the point where $\mathbf{S}^B_{p1}$, $\mathbf{S}^B_{p2}$ and $\mathbf{S}^B_{p3}$ intersect. Furthermore, for $j = 2, 4$,

$$\alpha_{3j} \neq 0, \quad h_j = h^B_{p1} \sin^2 \alpha_{3j} \neq 0, \quad p_{3j} = h^B_{p1} \sin \alpha_{3j} \cos \alpha_{3j}. \tag{2}$$

It may be proven that any $3I$ system lying within a $4G$ system must meet the following requirements (where $h^B_{p3} = 0$ is enforced, Fig. 3):

(i) $\mathbf{S}^B_{p1} = \mathbf{S}_{p1}$, i.e. $\ell^B_{p1} \equiv \ell_{p1}$ and $h^B_{p1} = h^B_{p2} = h_{p1}$, with $\mathbf{S}_{p1}$ being the principal screw normal to ℓ_{p3} with the highest pitch in absolute value, i.e. $|h_{p1}| \geq |h_{p2}|$;

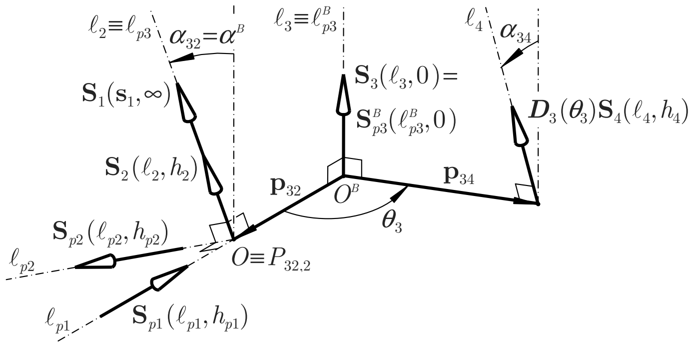

Fig. 4 Ternary generator of a 4-PSS with a constant class of the general type.

(ii) the center O^B of B must lie on ℓ_{p1}, namely $O^B - O = r^B \mathbf{s}_{p1}$, with $r^B \in \mathbb{R}$;
(iii) $\cos^2 \alpha^B = (h_{p2} - h^B_{p3})/(h_{p1} - h^B_{p3}) = h_{p2}/h_{p1}$, with α^B being the angle that ℓ^B_{p3} forms with ℓ_{p3}, evaluated according to the right-hand rule about $\mathbf{s}_{p1}$ and such that $-\pi/2 < \alpha^B \leq \pi/2$;
(iv) $r^B = (h_{p1} - h^B_{p3}) \sin \alpha^B \cos \alpha^B = h_{p1} \sin \alpha^B \cos \alpha^B$;
(v) h_{p1} and h_{p2} must have the same sign (when they are different from zero), i.e. $h_{p1} \geq h_{p2} \geq h^B_{p3} = 0$ or $h_{p1} \leq h_{p2} \leq h^B_{p3} = 0$.

For the sake of brevity, the proof of the above statements is not reported. It emerges from statements (i)–(v) that a $4G$ system may comprise only two $3I$ systems, i.e. B and B', such that $h^B_{p3} = 0$ (Fig. 3). B and B' are symmetric under reflection in ℓ_{p3}, and they coalesce when $h_{p2} = 0$ (in which case, $\alpha^B = \pi/2$ and $r^B = 0$).

If $S(\Theta)$ has to be a $4G$-PSS, h_{p1} and h_{p2} must remain constant as Θ varies. Hence, α^B must remain constant too. Two cases need to be distinguished, depending on whether $\mathbf{S}_1$ has infinite or finite pitch. If $h_1 = \infty$, $\mathbf{S}_1$ must be parallel to ℓ_{p3}, as the only ∞-pitch screw of S lies in $\mathscr{C}(\ell_{p3})$. Since $\ell^B_{p3} \equiv \ell_3$, α^B must thus coincide with the angle α_{31} between $\mathbf{s}_1$ and $\mathbf{s}_3$. Since $\cos \alpha_{31} = \mathbf{s}_1 \cdot \mathbf{s}_3 = \cos \alpha_{32} \cos \alpha_{21} - \sin \alpha_{32} \sin \alpha_{21} \cos \theta_2$ and $\sin \alpha_{32} \neq 0$, α_B may be constant only if $\alpha_{21} = 0$, in which case $\alpha^B = \alpha_{31} = \alpha_{32}$. Hence, $\mathbf{S}_1$ and $\mathbf{S}_2$ are parallel, and they span $\mathscr{C}(\ell_{p3})$ (Fig. 4). This condition must be enforced also if h_1 is finite. In fact, $\mathbf{S}_1$ must be reciprocal, for arbitrary values of θ_2, to a screw $\mathbf{S}_{r1}$ of pitch $-h_{p1}$ passing through O^B and lying on a plane perpendicular to ℓ_3. It is not difficult to verify, by direct computation, that this may happen only if $\mathbf{S}_1$ and $\mathbf{S}_2$ are collinear, i.e. if they span $\mathscr{C}(\ell_{p3})$[1]. The 4-system illustrated in Fig. 4 is, thus, persistent. O coincides with $P_{32,2}$, $\mathbf{S}_{p1}$ lies along the common normal between $\mathbf{S}_2$ and $\mathbf{S}_3$, $\mathbf{S}_{p2}$ is orthogonal to ℓ_{p1} and ℓ_{p3}, and the following conditions (deriving from Eqs. (1)–(2) and statements (i)–(iii)) apply:

[1] Indeed, $\mathbf{S}_1$ may be reciprocal, for arbitrary values of θ_2, to a screw of pitch $-h_{p1}$ intersecting ℓ_3 at right-angle, even if $p_{32} = 0$, $\alpha_{32} = \pi/2$ and $h_2 = h_{p1}$, with $\mathbf{S}_{r1}$ being, in this case, aligned with $\mathbf{S}_2$. However, requiring $\mathbf{S}_1$ to be also reciprocal to $\mathbf{S}_{r2}$, i.e. a screw both perpendicular to $\mathbf{S}_{r1}$ and intersecting it, straightforwardly leads to the condition that $\mathbf{S}_1$ and $\mathbf{S}_2$ must be collinear. Explicit calculations are not reported for the sake of brevity.

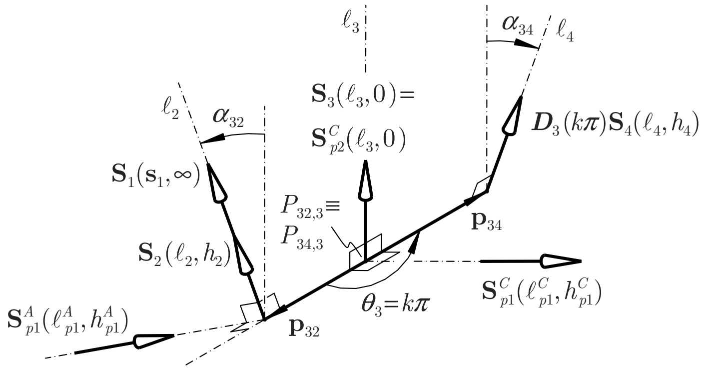

Fig. 5 Ternary generator of a 4*G*-PSS obtained from a 3*VIII*-PSS.

$$h_3 = 0, \quad h_4 = h_{p1} \sin^2 \alpha_{34}, \quad p_{34} = h_{p1} \sin \alpha_{34} \cos \alpha_{34}, \tag{3a}$$

$$p_{32} = h_{p1} \cos \alpha_{32} \sin \alpha_{32}, \quad h_{p2} = h_{p1} \cos^2 \alpha_{23}, \tag{3b}$$

with $h_{p1} \geq h_{p2} \geq 0$ or $h_{p1} \leq h_{p2} \leq 0$. In particular, when $h_{p2} = 0$ and $h_{p1} \neq 0$, Eq. (3b) implies $\alpha_{32} = \pi/2$ and $p_{32} = 0$. When $h_{p1} = 0$, Eqs. (3a)-(3b) require $h_{p2} = h_4 = p_{32} = p_{34} = 0$, and a ternary generator of a 4*I*-PSS is indeed obtained (cf. Fig. 2(c) in [1]).

The 4*G*-PSS in Fig. 4 may also be derived from the 3*VIII*-PSS generator shown in Fig. 7(a) of [3]. Let $\mathfrak{A} = \langle \mathbf{S}_1(\mathbf{s}_1, \infty), \mathbf{S}_2(\ell_2, h_2), \mathbf{S}_3(\ell_3, h_3) \rangle$, where $\mathbf{S}_1(\mathbf{s}_1, \infty)$ and $\mathbf{S}_2(\ell_2, h_2)$ form a cylindrical ISS $\mathscr{C}(\ell_2)$, and $\mathbf{S}_3(\ell_3, h_3)$ is a finite-pitch screw non-parallel to ℓ_2. According to [3], $\mathfrak{A}$ generates a 3*VIII*-PSS *A* such that

$$h^A_{p2} = h_3 + p_{32} \cot \alpha_{32}, \tag{4}$$

where $\mathbf{S}^A_{p2}(\ell^A_{p2}, h^A_{p2})$ is the principal screw of *A* perpendicular to ℓ_2 and $\mathbf{n}_{32}$. Since *A* contains one ∞-pitch screw and $\mathfrak{S} = \langle \mathfrak{A}, \mathbf{S}_4(\ell_4, h_4) \rangle$ has to generate a 4*G*-PSS, $\mathbf{S}_4$ must have finite pitch, and it cannot be parallel to ℓ_3.

If θ_3 is the angle between $\mathbf{n}_{32}$ and $\mathbf{n}_{34}$, whenever $\theta_3 = k\pi$, $k \in \mathbb{N}$, ℓ_2, ℓ_3 and ℓ_4 are perpendicular to the same direction, i.e. $\mathbf{n}_{32} = \pm \mathbf{n}_{34}$ (Fig. 5). In such configurations $\mathscr{C}(\ell_2)$, $\mathbf{S}_3$ and $\mathbf{D}_3(k\pi)\mathbf{S}_4$ cannot be linearly independent, otherwise they would form a 4-system reciprocal to an ∞-pitch screw parallel to $\mathbf{n}_{32}$ (and this could not be a 4*G* system). Thus, these configurations must be singular and, therein, $\mathbf{D}_3(k\pi)\mathbf{S}_4$ must belong, for any *k*, to *A*. This implies $P_{32,3} = P_{34,3}$ and $h_3 = 0$. Moreover, since $\mathbf{D}_3(0)\mathbf{S}_4$ and $\mathbf{D}_3(\pi)\mathbf{S}_4$ are symmetric under reflection in ℓ_3, they must form a cylindroid *C* having one of its principal screws, say $\mathbf{S}^C_{p2}$, on ℓ_3 [7, ch. 3]. Since *C* belongs to *A* and no line in *A* other than ℓ_2 may contain more than one screw, $\mathbf{S}^C_{p2}$ must necessarily coincide with $\mathbf{S}_3$, so that $h^C_{p2} = h_3 = 0$. By letting $\mathbf{S}^C_{p1}(\ell^C_{p1}, h^C_{p1})$ be the other principal screw of *C*, it must be [7, ch. 3]

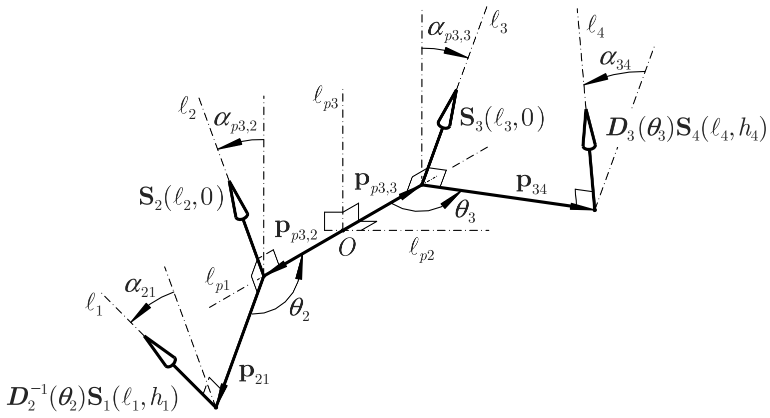

Fig. 6 Quaternary generator of a 4-PSS with a constant class of the general type.

$$h_4 = h^C_{p2} + (h^C_{p1} - h^C_{p2})\sin^2\alpha_{34} = h^C_{p1}\sin^2\alpha_{34}, \tag{5a}$$

$$p_{34} = (h^C_{p1} - h^C_{p2})\sin\alpha_{34}\cos\alpha_{34} = h^C_{p1}\sin\alpha_{34}\cos\alpha_{34}. \tag{5b}$$

By requiring $\mathbf{S}^C_{p1}$ to belong to A, one also obtains that

$$h^A_{p2} = h^C_{p1} + p_{32}\cot(\alpha_{32} + \pi/2) = h^C_{p1} - p_{32}\tan\alpha_{32} \tag{6}$$

and, thus, recalling Eq. (4) and rearranging terms,

$$p_{32} = h^C_{p1}\sin\alpha_{32}\cos\alpha_{32}, \quad h^A_{p2} = h^C_{p1}\cos^2\alpha_{32}. \tag{7}$$

By varying θ_3, $\mathbf{S}_3$ and $\mathbf{D}_3(\theta_3)\mathbf{S}_4$ generate a pencil of cylindroids all congruent to C, i.e. a 3I-PSS with principal pitches equal to 0 and h^C_{p1}. By letting $h^C_{p1} = h_{p1}$ and $h^A_{p2} = h_{p2}$, Eqs. (5) and (7) coincide with Eqs. (3a)-(3b). The generator in Fig. 4 is evidently re-obtained.

2.2 The quaternary generator

The arguments developed in Section 2.1 provide a clue for obtaining a quaternary generator of 4G-PSS. It has been seen, in fact, that a 4G system such that $h_{p1} > h_{p2} \geq 0$ or $h_{p1} < h_{p2} \leq 0$ comprises only two 3I systems with a 0-pitch central principal screw, i.e. B and B' (Fig. 3). According to statements (ii)–(iv), the poses of the central axes ℓ^B_{p3} and $\ell^{B}_{p3}{}'$ of B and B' are unambiguously determined by the value of the principal pitches of S, i.e. h_{p1} and h_{p2}. In Fig. 4, the screws $\mathbf{S}_3$ and $\mathbf{D}_3(\theta_3)\mathbf{S}_4$ span (as θ_3 varies) one of these 3I systems, say B, with the entire 4-system being generated by composing B with the cylindrical ISS $\mathscr{C}(\ell_{p3})$. If $\mathscr{C}(\ell_{p3})$ is replaced by two screws, i.e. $\mathbf{S}_2(\ell_2, 0)$ and $\mathbf{D}_2^{-1}(\theta_2)\mathbf{S}_1(\ell_1, h_1)$, which span (as θ_2 varies) the other $3I$ system, i.e. B', the same vector subspace is obviously obtained (Fig. 6). The joint screws of the described generator satisfy the conditions

$$h_2 = h_3 = 0, \quad \alpha_{p3,2} = \alpha_{p3,3}, \tag{8a}$$

$$h_{p2} = h_{p1}\cos^2\alpha_{p3,2}, \quad p_{p3,2} = p_{p3,3} = h_{p1}\cos\alpha_{p3,2}\sin\alpha_{p3,2}, \tag{8b}$$

$$h_1 = h_{p1}\sin^2\alpha_{21}, \quad p_{21} = h_{p1}\sin\alpha_{21}\cos\alpha_{21}, \tag{8c}$$

$$h_4 = h_{p1}\sin^2\alpha_{34}, \quad p_{34} = h_{p1}\sin\alpha_{34}\cos\alpha_{34}, \tag{8d}$$

where $p_{p3,i}$ and $\alpha_{p3,i}$ are, respectively, the shortest distance and the relative angle between ℓ_i and ℓ_{p3}, $i = 2, 3$. The quaternary generator in Fig. 6 does not allow $h_{p2} = 0$ and $h_{p1} \neq 0$, since in this case $\alpha_{p3,2} = \alpha_{p3,3} = \pi/2$ and $p_{p3,2} = p_{p3,3} = 0$, and $\mathbf{S}_2$ and $\mathbf{S}_3$ would coincide. Also, this generator may not degenerate into a $4I$-PSS, as for $h_{p1} = 0$ all screws would have zero pitch and pass through O.

3 Conclusions

Two generators of 4-dimensional screw systems with a constant class of the general type (i.e. $4G$-PSSs) were disclosed. It may be proven that no other $4G$-PSS generators exist. Due to space limitations, the proof is omitted, but it will be reported in a future extended version of the contribution.

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