Self Dual Topology of Parallel Mechanisms with Configurable Platforms

Patrice Lambert and Just L. Herder

Abstract This paper presents first an analysis of the topology of mechanisms via Graph Theory and Screw Theory and next the principle of dual mechanisms in terms of their mobility and overconstraints. Using dual graphs, the graph representations of the mechanisms that are dual to hybrid and Delta mechanisms are revealed. The concept of parallel mechanisms with configurable platforms (PMCPs) is introduced and it is shown that the graph reduction of PMCPs always results in a wheel graph, which has the interesting property of being self-dual. In case of self dual topology, it is then possible to directly convert any method developed for their mobilty analysis into an overconstraint analysis method and vice versa. This self dual topology property can also be exploited to create new PMCPs and is an important aspect in the future development of a type synthesis method that includes PMCPs.

Key words: Parallel Mechanisms, Configurable Platform, Topology, Mobility, Overconstraints, Duality

1 Introduction

A pure parallel mechanism is formed by two rigid links, called the base and the end-effector, connected in parallel by independent serial chains, called legs. The concept behind parallel mechanisms with configurable platforms (PMCPs) is that the rigid (non-configurable) end-effector is replaced by a closed-loop chain (the configurable platform), see for example Fig. 1. Some of the links of this closed-loop chain are attached to the legs so its configuration can be fully controlled from the motors located near the base. The use of a closed-loop chain instead of a rigid

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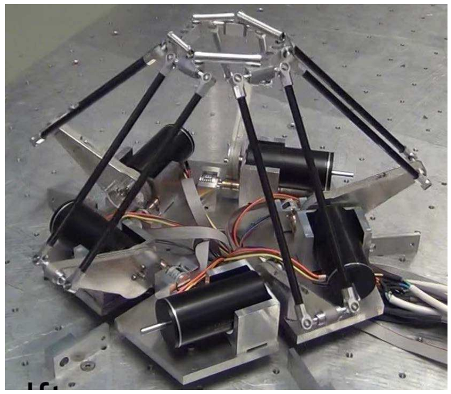

Fig. 1 The 5 DOF PentaG robot with configurable platform. The configurable platform can translate in 3 directions and has 2 internal DOF, providing additional rotation and grasping capabilities. This architecture was used to develop both a haptic device and a pick-and-place robot.

platform creates multiple end-effectors that can be used for example to add grasping capabilities. In this case the robot can combine motions and grasping into a structure that provides an inherent high structural stiffness, since no motors are needed at the end-effector location to provide the grasping and all motors can be grounded on the base. Very few PMCPs have been presented in the literature [5, 7]. Mohamed and Gosselin [6] proposed a first generalization of the concept of both planar and spatial PMCPs. In their article, all the mechanisms they proposed were not overconstrained and the case of overconstrained PMCPs was not addressed. PMCPs are ignored in type synthesis methods of mechanisms such as in [3, 2]. In order to describe a topological relation between mobility and overconstraints, Davies [1] introduced the idea of dual mechanisms with the use of dual graphs. In his article, he presents the dual mechanism of a four-bar linkage and a self-dual truss structure.

This article presents a topology analysis of PMCPs via Graph Theory and Screw Theory and shows that unlike pure parallel mechanisms, the topology of PMCPs is represented by a wheel graph, which has the remarkable property of being self-dual. The self dual topology of PMCPs is then exploited to extend the mobility analysis method presented in [4] directly into an overconstraint analysis method and is also used to create new overconstrained PMCPs.

2 Duality Between Parallel Mechanisms and Single Closed-Loops

Closing the mechanical loops of a parallel mechanism introduces dependencies between the joint velocities of the chains and can also produce internal stresses due to overconstraints of the assembly. The mobility and constraints of a serial chain are here expressed via Screw Theory. For each n-twist system S representing the mobility of a serial chain, it is possible to obtain a reciprocal $(6-n)$-wrench system $S^{\perp}$ representing its constraints. For a parallel mechanism with k legs, in which the mobility of each leg i is represented by the twist system T_i, the twist system T_M representing the instantaneous mobility between the base and the end-effector is given by the intersection of the mobility of all legs:

$$T_M = \bigcap_{i=1}^{k} T_i \tag{1}$$

Internal stresses in parallel mechanisms are due to redundant constraints between the base and the end-effector. For a parallel mechanism with $i = 1..k$ legs, in which the constraint system of each leg is represented by the wrench system W_i, a set of $(k-1)$ wrench systems W_{Rj} representing the independent overconstraints is obtained by calculating the intersection of the wrench system W_j of leg j with the summation of the constraints of the legs previously connected to the end-effector.

$$W_{Rj} = W_j \bigcap \left(\sum_{i=1}^{j} W_i \right) \quad \text{for } j = 2..k \tag{2}$$

For any mechanism, the solution for the mobility must respect the condition that the sum of the finite twists (describing the joint velocities) of all joints that belong to the same closed loop is zero. In a dual way, the solution for the overconstraints must respect the condition that the sum of the finite wrenches (describing the internal stresses) of all chains that are connected to the same rigid link is zero. The particular way to obtain those solutions depends on the topology of the mechanism, i.e. the particular arrangement of the closed loops that form the mechanism. Graph Theory is the study of pairwise relations between objects and since each joint or each serial chain of joints in a mechanism connects strictly two links, it can be used to analyse the topology of mechanisms. Any mechanism can be represented with a corresponding graph in which a vertex represents a rigid link and an edge represents a joint or a serial chain of joints. The duality between the mobility and overconstraint conditions, which both require that the sum of a set of finite screws is zero, will be investigated more deeply with the use of dual graphs.

The dual graph B of an original graph A is a graph in which each vertex of B corresponds to a loop of A and vice-versa. Edges that are incident to a vertex in the original graph are in the same direction as the corresponding loop of the dual graph and vice versa. If the twist systems of the dual edges are defined as the reciprocal of the twist systems of the original edges, the mechanism corresponding to the dual graph is dual to the original mechanism. The dual mechanism of a pure parallel mechanism is always a single closed-loop and vice versa. Fig. 2 shows an example of the graph representation of a pure parallel mechanism with four legs. In [1], Davies used the concept of dual graphs to show that the dual mechanism of a planar four-bar linkage is a parallel mechanism with four parallel SU legs. The principle of dual mechanisms is here extended to some mechanisms that have a topology that is not purely parallel. Following this principle, the dual graphs of a hybrid and a Delta mechanism are shown for the first time in Fig 3.

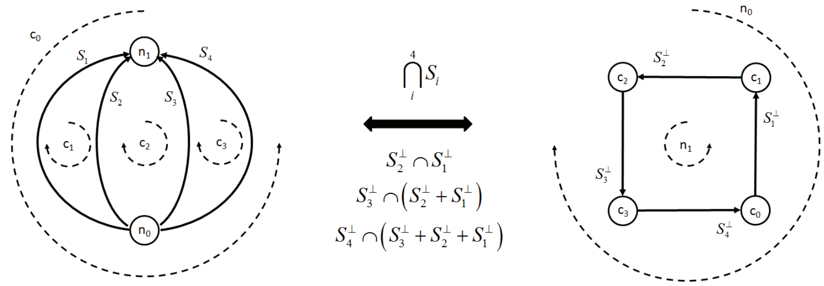

Fig. 2 Graph representation of a parallel mechanism with four legs and its dual single closed-loop. The screw system over the big arrow represents the mobility of the parallel mechanism and the overconstraints of the single closed-loop. The screw systems below the arrow represent the overconstraints of the parallel mechanism and the mobility of the single closed-loop.

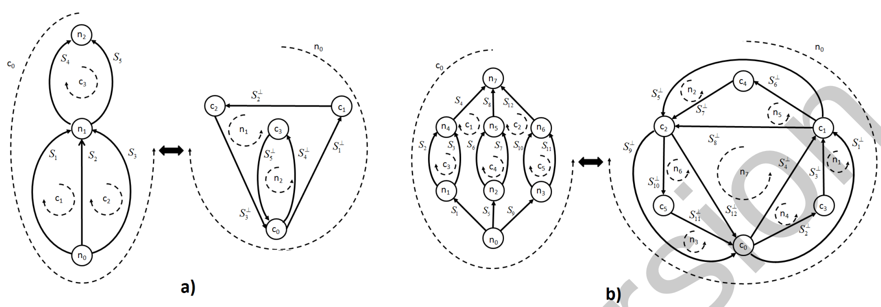

Fig. 3 Graph representation of a) a hybrid mechanism and b) a Delta mechanism and their corresponding dual mechanisms. In both cases, the mobility of the dual mechanism corresponds to the overconstraints of the original mechanism and vice versa.

3 Self Dual Topology of Parallel Mechanisms with Configurable Platforms

The novel concept behind parallel mechanisms with congurable platforms is that the rigid (non-configurable) end-effector is replaced by a closed-loop chain (the configurable platform). The use of a closed-loop chain instead of a rigid platform allows robots based on such an architecture to have multiple end-effectors on the platform while all the motor are located on the base. The graph reduction of a PMCP is always a wheel graph, in which the center of the wheel represents the base, the spokes of the wheel represent the legs and the rim represents the configurable platform. Wheel graphs have the interesting property of being self-dual. Fig. 4 shows the wheel graph of a PMCP with 4 legs and its dual graph. It follows that each PMCP has a dual PMCP for which the mobility of the dual PMCP corresponds to the overconstraints of the original PMCP and vice versa. In particular, the method presented in [4] to calculate the distribution of the mobility of overconstrained PMCPs can be

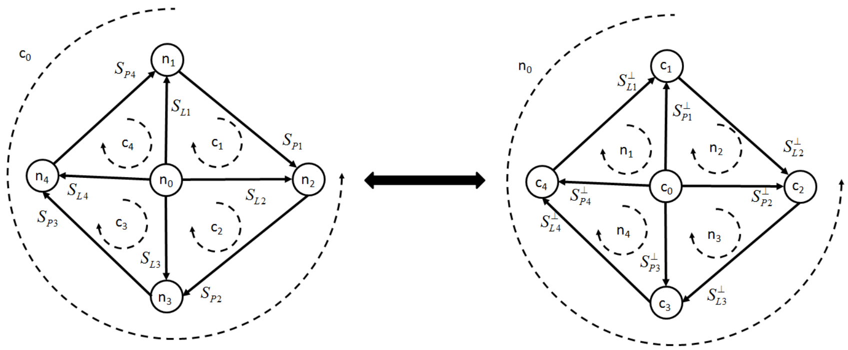

Fig. 4 Self dual wheel graphs of a parallel mechanism with configurable platform and 4 legs

directly used to calculate the distribution of the overconstraints using the dual graph as input. It should be noted that in the dual graph representation of Fig. 4, the edges that originaly represented the platform are now representing the legs and vice versa, and that the reciprocal screw system is used to describe the mobility of each dual edge.

4 Example

This section shows an example of two PMCPs that are dual to each other. The original mechanism M1 is shown in Fig. 5 a). It has four legs L1, L2, L3 and L4 and each leg consists of three parallel joints. The configurable platform is a single closed loop with 12 parallel joints. Two adjacent end-effector links n_i are connected by platform limbs P1, P2, P3 or P4. In order to create the dual mechanism, we first need to express the twist system of each leg and each platform limb. Using a reference frame located in the center of the base, the twist systems are

$$T_{L1} = T_{L3} = \left\{ \begin{bmatrix} \mathbf{0} \\ \mathbf{x} \end{bmatrix}, \begin{bmatrix} \mathbf{0} \\ \mathbf{z} \end{bmatrix}, \begin{bmatrix} \mathbf{y} \\ \mathbf{0} \end{bmatrix} \right\} \ \& \ T_{L2} = T_{L4} = \left\{ \begin{bmatrix} \mathbf{0} \\ \mathbf{y} \end{bmatrix}, \begin{bmatrix} \mathbf{0} \\ \mathbf{z} \end{bmatrix}, \begin{bmatrix} \mathbf{x} \\ \mathbf{0} \end{bmatrix} \right\} \quad (3)$$

$$T_{Pi} = \left\{ \begin{bmatrix} \mathbf{0} \\ \mathbf{x} \end{bmatrix}, \begin{bmatrix} \mathbf{0} \\ \mathbf{y} \end{bmatrix}, \begin{bmatrix} \mathbf{z} \\ \mathbf{0} \end{bmatrix} \right\} \ \text{for} \ i = 1..4 \quad (4)$$

Unlike pure parallel mechanisms, PMCPs have multiple end-effectors, represented by the leg attach points n_i. Since the graph representation of such a mechanism is not a series-parallel graph, their global mobility can not be calculated using the traditional rules of addition of twist systems for joints in series and intersection of twist systems for legs in parallel. In [4], a method was proposed to calculate the mobility of mechanisms that have a non-series-parallel graph that is particularly

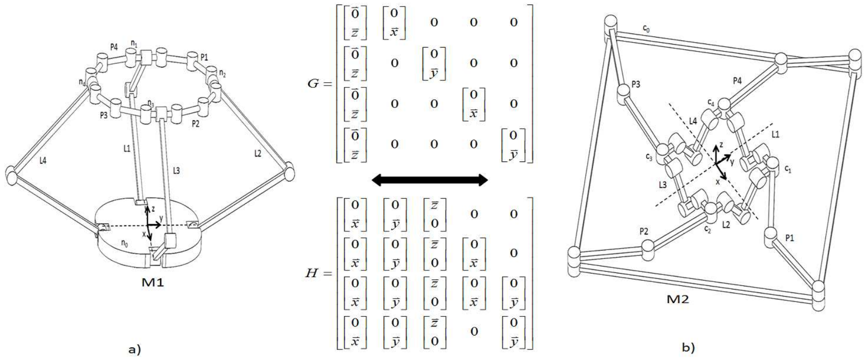

Fig. 5 Dual mobility and overconstraints of two PMCPs. G_L represents the mobility of M1 and the overconstraints of M2. H_P represents the mobility of M2 and the overconstraints of M1. Each line of a matrix of mobility uses twist screws to represent the mobility of a leg end-effector n_i or c_i relatively to the base. Each line of a matrix of overconstraints uses wrench screws to represent the internal stresses in a platform limb between two end-effectors.

suitable for PMCPs. The solution given by the method forms a matrix in which row i represents the mobility of an end-effector n_i relatively to the base and each column represents a global mobility of the mechanism. The final leg mobility matrix G of mechanism M1 obtained from this method is

$$G = \begin{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{z} \end{bmatrix} & \begin{bmatrix} \mathbf{0} \\ \mathbf{x} \end{bmatrix} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \begin{bmatrix} \mathbf{0} \\ \mathbf{z} \end{bmatrix} & \mathbf{0} & \begin{bmatrix} \mathbf{0} \\ \mathbf{y} \end{bmatrix} & \mathbf{0} & \mathbf{0} \\ \begin{bmatrix} \mathbf{0} \\ \mathbf{z} \end{bmatrix} & \mathbf{0} & \mathbf{0} & \begin{bmatrix} \mathbf{0} \\ \mathbf{x} \end{bmatrix} & \mathbf{0} \\ \begin{bmatrix} \mathbf{0} \\ \mathbf{z} \end{bmatrix} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \begin{bmatrix} \mathbf{0} \\ \mathbf{y} \end{bmatrix} \end{bmatrix} \tag{5}$$

The fact that the matrix G has five columns indicates that the mechanism has 5 DOF. The first column represents a motion where all legs move in the vertical direction. The remaining columns show that each leg can also move independently in the horizontal direction of the plane of their parallel joints. Since all PMCPs have a self dual topology, the overconstraints of mechanism M1 can be obtained by appling the same mobility analysis method to the dual PMCP. The original and dual graphs are shown in Fig.4 The dual graph gives the information about the way the dual chains must be connected in the dual mechanism. The reciprocal screw system of each serial chain must be calculated in order to obtain the dual serial chains. In this particular mechanism, each leg and each platform chain is formed by three parallel joints. Those chains are known to be self reciprocal and therefore the screw system representing their constraints is the same as the screw system representing their mobility.

$$S_{Li}^{\perp} = S_{Li} \;\;\&\;\; S_{Pi}^{\perp} = S_{Pi} \quad \text{for} \;\; i = 1..4 \tag{6}$$

We can now assemble the dual mechanism by connecting the dual chains according to the edges of the dual graph. The resulting mechanism M2 is shown in Fig. 5 b). The mobility of M2 is calculated using the same method that was used to calculate the mobility for M1. The resulting leg matrix of mobility H obtained is

$$H = \begin{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{x} \end{bmatrix} & \begin{bmatrix} \mathbf{0} \\ \mathbf{y} \end{bmatrix} & \begin{bmatrix} \mathbf{z} \\ \mathbf{0} \end{bmatrix} & \begin{bmatrix} \mathbf{0} \\ \mathbf{x} \end{bmatrix} & \begin{bmatrix} \mathbf{0} \\ \mathbf{y} \end{bmatrix} \\ \begin{bmatrix} \mathbf{0} \\ \mathbf{x} \end{bmatrix} & \begin{bmatrix} \mathbf{0} \\ \mathbf{y} \end{bmatrix} & \begin{bmatrix} \mathbf{z} \\ \mathbf{0} \end{bmatrix} & \begin{bmatrix} \mathbf{0} \\ \mathbf{x} \end{bmatrix} & \begin{bmatrix} \mathbf{0} \\ -\mathbf{y} \end{bmatrix} \\ \begin{bmatrix} \mathbf{0} \\ \mathbf{x} \end{bmatrix} & \begin{bmatrix} \mathbf{0} \\ \mathbf{y} \end{bmatrix} & \begin{bmatrix} \mathbf{z} \\ \mathbf{0} \end{bmatrix} & \begin{bmatrix} \mathbf{0} \\ -\mathbf{x} \end{bmatrix} & \begin{bmatrix} \mathbf{0} \\ -\mathbf{y} \end{bmatrix} \\ \begin{bmatrix} \mathbf{0} \\ \mathbf{x} \end{bmatrix} & \begin{bmatrix} \mathbf{0} \\ \mathbf{y} \end{bmatrix} & \begin{bmatrix} \mathbf{z} \\ \mathbf{0} \end{bmatrix} & \begin{bmatrix} \mathbf{0} \\ -\mathbf{x} \end{bmatrix} & \begin{bmatrix} \mathbf{0} \\ \mathbf{y} \end{bmatrix} \end{bmatrix} \tag{7}$$

This mechanism also has 5 DOF. The first three columns show that the configurable platform can move in the XY plane and rotate around the Z direction. The fourth column shows that legs P1 and P2 can move relatively to leg P3 and P4 in the X direction and the fifth column shows that legs P1 and P4 can move in the Y direction relatively to legs P2 and P3. Since these two mechanisms are dual, the mobility of M1 corresponds to the overconstraints of M2 and vice-versa. The matrix H is the platform matrix of overconstraints of M1. In a platform matrix of overconstraints, each line represents the internal forces transmited by a platform chain and each column represents an independent overconstraint.

The overconstraints of both mechanisms are interpreted as follows: The first three columns of H represent the 3 planar overconstraints of the configurable platform of M1. Column 4 represents internal stresses that occur in the platform limbs when leg L1 and leg L3 of M1 are not perfectly oriented about the axis X. Column 5 represents the internal stresses that occur in the platform limbs when leg L2 and leg L4 are not perfectly oriented around the axis Y. The overconstraints of mechanism M2 are represented by the platform matrix of overconstraints G. The first column represents internal stresses around the Z axis that occur in all the platform limbs if the sum of the angle between the platform limbs is not 360 degrees. The second column of G shows that the platform limb L1 of M2 must be perfectly oriented around the X axis in order to be connected between leg P4 and leg P1. Columns 3, 4 and 5 of G shows similar overconstraint conditions for platform limbs L2, L3 and L4 of mechanism M2.

5 Conclusion

This paper presented first the principle of dual mechanisms in terms of mobility and overconstraints for some mechanisms that have a topology that is not purely parallel. The graph and screw representations of the mechanisms that are dual to hybrid and Delta mechanisms were revealed. It was explained that the graph reduction of PMCPs always result in a wheel graph, which has the interesting property of being self-dual. Unlike most other classes of mechanism, it is possible to directly apply any method developed for the mobilty analysis of PMCPs to their overconstraint analysis and vice versa, thanks to their self-dual topology. This self-dual topology property can also be used to generate new PMCPs using the dual mechanism of original PMCPs. PMCPs are promising solutions for robot architecture since they can operate multiple end-effectors while all motors are located at the base. Applications include interaction with humans or environment, in haptic devices and grasping applications, respectively. They are currently ignored by type synthesis methods. A better understanding of the fundamentals of their mobility and overconstraints will help robot designers to consider them as a valid option in their choice of a robot architecture and is an important aspect in the future development of a broader type synthesis method that includes mechanisms with this topology.

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