A Novel Mechanism with Redundant Elastic Constraints for an Actual Revolute Joint

Delun WANG, Zhi WANG, Huimin DONG, and Shudong YU

Abstract A novel spatial mechanism with redundant elastic constraints is presented in this paper to establish a comprehensive model for simulating kinematic characteristics of an actual revolute joint with flexibility and geometric errors. The rigid cam profiles are specified to represent the geometrical errors. Elastic springs are used to simulate the deformations of joint components and their surfaces for the actual machine parts. The proposed RE mechanism, consisting of suspended cams and multiple followers with springs, yields a total of 32 basic equations for displacement analysis. The numerical results obtained using the proposed approach were compared with the experimental data for an example revolute joint; good agreement was achieved for joint kinematic characteristics. The proposed approach provides a new application of the theory of mechanism in comprehensive performance analysis of a complex mechanical system having many components with machining errors.

Key words: Elastic constraint, Error, Mechanism, Revolute joint.

1 Introduction

In a kinematical analysis of mechanism, the links are often assumed to be rigid, the joints are assumed to have ideal geometries and maintain rigid under loads. Take a revolute joint as an example. The shaft in the revolute joint is a rigid and perfect cylinder, which can rotate only about its ideal axis of rotation regardless of the applied loads. In reality, all components deform under loads, the shapes of

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these components have geometrical errors due to machining. Therefore, the shaft in an actual revolute joint has one dominating rotational DOF and five additional DOF's due to joint machining errors and component flexibility. We believe that the kinematic characteristics of a joint in machine design can be obtained by studying the kinematics of a corresponding spatial mechanism with appropriate constraints when considering machining errors and joint flexibility[1-2].

In this paper, our scope is focused on the analysis of the complex movement of the shaft in a revolute joint with an aim to addressing the effects of machining errors and flexibility (or simply revolute precision) on performance of an actual revolute joint. The revolute precision is a key noteworthy performance of machine tools and various precision instruments. The kinematical analysis and synthesis of an actual revolute joint lays a solid theoretical basis for machine precision design[3-5].

2 An Actual Revolute Joint with Elastic Constraint and Error

An actual revolute joint consists of a rotating shaft, two or more bearings, and a housing unit. The shaft can rotate freely about its axis, as shown in Fig 1.

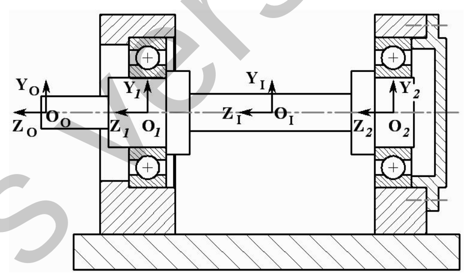

Fig. 1 A typical construction of actual revolute joint

Geometrical errors: Every component is manufactured with geometrical errors. Because of the presence of geometrical errors, the shaft no longer has a perfectly cylindrical surface. Similarly, the actual bearing components such as the outer and inner races also deviate from their ideal nominal surfaces. These components with various types of geometrical errors are assembled together to form an actual revolute joint to accommodate the rotational motion of the shaft.

Elasticity and deformations: Every component in an actual revolute joint is made of isotropic or anisotropic materials with finite elasticity. Under loads, these components deform. The load-induced component deformations magnify the machining-induced geometrical errors. The actual geometrical shapes of the shaft and bearings also vary with loads.

Constraints and motion: The shaft in an actual revolute joint, supported and constrained by bearings, rotates about the Z-axis freely. The additional five elastic DOF's, two rotations about the X-axis and the Y-axis, and three translations along the X, Y, and Z directions, are always present when geometrical errors and component flexibility are taken into consideration. As a result, a geometrical element of the shaft in an actual revolute joint has six DOF's.

3 The RE Mechanism for an Actual Revolute Joint

Geometry and motion equivalence: Each component has its own geometrical errors, which affects the positions and kinematical characteristics of the shaft. In the motionless conjunction planes (MLCP), the components are fixed together. The components deform with assembled loads, their shapes and positions will also change. While in the motional conjunction surfaces (MCS), the components can only move relatively. The kinematic characteristics (positions and loci) of the motional components will change due to the geometrical errors. This means that the geometrical errors of one component may be transferred into those of another component by the MLCP or the MCS. The shaft in an actual revolute joint is regarded as cams with specified rigid profiles and suspended by the translating followers as shown in Fig 2. The shaft motion is equivalent to that of the cam having a profile equivalent to the geometrical dimensions and their error characteristics.

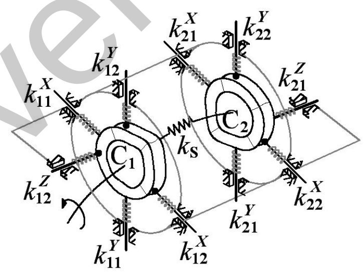

Fig. 2 The RE mechanism with redundant elastic constraints for an actual revolute joint

Elasticity equivalence: The surfaces of components for an actual revolute joint experience deform under loads. In general, the displacements of the components due to elastic deformations may be transferred to the surface of the shaft by the MLCP or the MCS. As a result, the shaft will have additional elastic displacements, which are determined by the component flexibility and loads. The elasticity of the constraint is equivalent to several springs attached to the followers of the mechanism[6].

RE mechanism: A mechanism, with redundant degree of freedom and elastic constraints, can be defined as an RE mechanism, where R stands for the redundant degree of freedom and E denotes elastic constraints. In an RE mechanism, both rigid links and elastic components (springs) are simultaneously present; the geometrical characteristics and physical characteristics can be extensively described. Particularly, the redundant DOF's coexist with elastic constraints.

In Fig 2, the RE mechanism has a total of 12 links, a shaft with double cams, 10 translational followers and a base frame. Each follower forms a 1-DOF prismatic pair with the base frame, and a 5-DOF pair with the cam. The number of degrees of freedom can be calculated by the Gruebler's equation[7]

$$F = 6(n-g-1) + \sum_{i=1}^{g} f_i \tag{1}$$

where n (=12) is the number of links; g (=20) is the total number of kinematic pairs; $f_1(=1\times10)$; $f_2(=5\times10)$. The number of DOF's is F=6. These 6 DOF's are the rotation of the cam about its axis and the five elastic DOF's mentioned above. In addition, the RE mechanism has another 10 DOF's associated with the 10 translational followers, in which five of them correspond to the elastic constraint of the cam, and the other five are the elastic motion of the cam with redundant DOF's. It is enough for a rigid cam mechanism to be constrained with 5-DOF motion. We can see the significance of the elastic motions with redundant DOF's.

The RE mechanism in Fig 2 is a simplest mechanism with redundant DOF's and elastic constraints for the actual revolute joint with the two bearings. It will keep the same DOF's if an additional follower is introduced since it is accompanied by a 1-DOF pair and a 5-DOF pair, which counterbalances the six DOF's of the introduced follower. This performance provides convenience for displacement analysis of the actual revolute joint with construction error and over-constraint constructions.

Discussion: In mechanism, the constraints and DOF's are closely related. The DOF is the number of independent parameters required to define configurations of the mechanism, with respect to the base frame. For an actual revolute joint, a point of the base link (component) experiences elastic displacements under loads. Contrary to the rigid mechanisms, the constraining link or the follower in an RE mechanism has the elastic motion of springs. Therefore, the DOF in an RE mechanism has a different meaning from that in mechanism with rigid links only.

4 The Basic Equations of an RE Mechanism

For displacement analysis, the basic equations of an RE mechanism have to be determined, including the geometrical equations, the equations of equilibrium, and the equations of physical properties. The geometrical equations can be given as following according to the construction of the RE mechanism, shown in Fig 3.

$$|\mathbf{r_i} + \mathbf{r_{ij}^V} + (\mathbf{L_{ij}^V} - \boldsymbol{\delta}_{\mathbf{ij}}^{\mathbf{V}})| = L_{Di}/2 \quad i,j = 1,2; V = X,Y \tag{2}$$

$$|\mathbf{r_i} + \mathbf{r_{ij}^Z} + (\mathbf{L_{ij}^Z} - \boldsymbol{\delta}_{\mathbf{ij}}^{\mathbf{Z}})| = L_{Bi} \quad i = 1, j = 2 \;\; or \;\; i = 2, j = 1 \tag{3}$$

The equations of equilibrium for an RE mechanism may firstly be written as

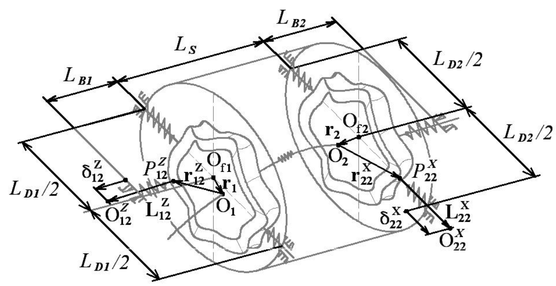

Fig. 3 The displacements of the RE mechanism

$$\sum_{i=1}^{2}\sum_{j=1}^{2}(\mathbf{F_{ij}^X}+\mathbf{F_{ij}^Y}+\mathbf{F_{ij}^Z})+\mathbf{F_I}+\mathbf{F_O}=\mathbf{0} \tag{4}$$

$$\sum_{i=1}^{2}\sum_{j=1}^{2}(\mathbf{F_{ij}^X P_{ij}^{Xf}}+\mathbf{F_{ij}^Y P_{ij}^{Yf}}+\mathbf{F_{ij}^Z P_{ij}^{Zf}})+\mathbf{F_I r_I}+\mathbf{F_O r_O}+\mathbf{M_I}+\mathbf{M_O}=\mathbf{0} \tag{5}$$

The equations of equilibrium for the followers are $|\mathbf{F_{ij}^{VK}}| = |\mathbf{F_{ij}^{V}}|\cos\alpha_{ij}^{V}$.

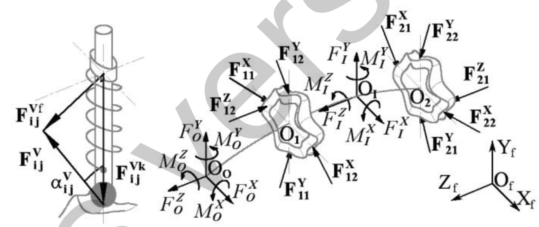

Fig. 4 The forces and moments in the RE mechanism

The equations of physical properties in an RE mechanism can be written from the elasticity of the follower springs as equation (6). And the physical equations of shaft connected to two cams as equation (7)

$$\mathbf{F_{ij}^{VK}} = k_{ij}^{V}\delta_{\mathbf{ij}}^{\mathbf{V}} \quad i,j=1,2;V=X,Y,Z \tag{6}$$

$$\mathbf{K_S}[\Delta_{12}^X,\Delta_{12}^Y,\Delta_{12}^Z,\theta_{12}^X,\theta_{12}^Y,\theta_{12}^Z]^T=\mathbf{P} \tag{7}$$

There are six parameters to be determined in Equation (7), which describes the displacements of the shaft $(\Delta_{12}^X,\Delta_{12}^Y,\Delta_{12}^Z,\theta_{12}^X,\theta_{12}^Y,\theta_{12}^Z)$, or the relative differences between the two cams.

Equation (2) to (7), totaling 32 equations with 32 variables, are called the basic equations of an RE mechanism. From the basic equations, the positions and postures of two cams with rotation angle θ_z can now be located at any instant. As a result, any point of the shaft will produce a spatial curve in the based frame, which is the actual non-circular trajectory. The precision of an revolute joint, just the error between the actual trajectory and the ideal curve, can be evaluated. The variations of precision with the working conditions of an actual revolute joint can also be exposed.

5 Case Study

An RE mechanism can be constructed for the actual revolute joint shown in Fig 1. The basic parameters of the revolute joint are given in Table 1.

Table 1 The construction parameters of the actual revolute joint

Bore (d)	60.0 mm	Ball diameter (D_b)	22.22 mm
Outside diameter (D)	130.0 mm	Distance between O_1 and O_O	130.0mm
Inner ring width (B)	31.0 mm	Distance between O_1 and O_I	105.0mm
Outer ring width (C)	31.0 mm	Distance between O_1 and O_2	160.0mm

Parameters of elastic constraints: In this mechanism, there are 10 follower springs. The deformations of the springs are intended to be equivalent to the elastic displacement of the shaft in the revolute joint, which are caused by the combined elastic deformation of the bearings, bearing blocks, flanges and the shaft. For convenience, we calculate the stiffness of the bearing and the supporting structure individually as the rolling bearing is a standard part.

The stiffness of bearings can be calculated by the approximate formula [8]. The stiffness in the radial direction is $k_{ij}^{BV} = 1190282\delta_r^{1/2}$(i,j =1,2;V=X,Y), and in the axial direction, the stiffness is $k_{ij}^{BV} = 46568\delta_r^{1/2}$(i,j =1,2; V = Z).

Table 2 Stiffness of the supporting structures ($\times 10^5 N/mm$)

k_{11}^{SX}	k_{12}^{SX}	k_{11}^{SY}	k_{12}^{SY}	k_{11}^{SZ}	k_{12}^{SZ}
8.50	8.50	6.06	10.1	unconstrained	2.33
k_{21}^{SX}	k_{22}^{SX}	k_{21}^{SY}	k_{22}^{SY}	k_{21}^{SZ}	k_{22}^{SZ}
5.64	5.64	4.45	7.27	1.19	unconstrained

The stiffness of the supporting structures, including the bearing block, flange and shaft, is calculated by using the finite element method (FEM) in this paper . The results are given in Table 2. The stiffness of the shaft connected to the two cams is calculated by FEM according to equation (7).

Geometrical feature and cam profiles: The geometric features and the kinematic characteristics of the cams are similar to those of the bearing inner rings. Ideally, the cam profile is the ideal profile of the inner race ($r_0(\theta, z)$). In fact, the geometric features and kinematic characteristics of the bearing inner rings are related to the geometric errors and loads.

Each cam follower of the RE mechanism corresponds to a specified cam profile. With the MLCP and the MCS, the geometrical errors are transferred into positional errors ($\delta(\theta, z)$) of the moving component. The ideal profile of the inner race ($r_0(\theta, z)$) is designated as the initial profile of the cams, and then, the positional errors ($\delta(\theta,z)$) and the elastic coefficient ($\varepsilon(\theta,z)$) are taken into account. The profiles equation of the cams can be written as

$$r = S(\theta,z) = r_0(\theta,z) + \delta(\theta,z) + \varepsilon(\theta,z) \tag{8}$$

The actual profiles of the bearing components are measured respectively. Based on the elastic deformation, the geometrical errors may change the geometrical shape of the outer and inner races by the interference fitting between the shaft and inner ring, which can be calculated by using a contact model and converted into the positional errors ($\delta(\theta,z)$)of the kinematic model in the RE mechanism.

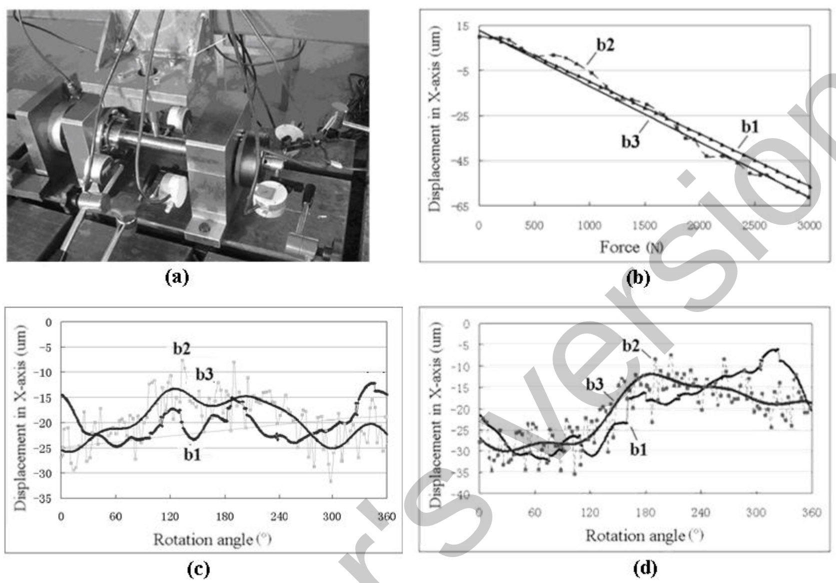

Fig. 5 Experiment of an actual revolute joint

Experiment: To compare the results of displacement analysis of the RE mechanism with that of experiments, the test equipment is set up for the actual revolute joint, shown in Fig 5(a). The displacements of the reference point of the shaft, which is located at the cross-section of the outer section, are measured in the X-direction. The curve b1 is the numerical results computed by the model of RE mechanism, the curve b2 is the measuring results of the experiment and the curve b3 is the fitting curve of the curve b2, which are shown as followings.

1) Fig 5(b) shows the variations of the displacements with the driving force from 0 to 3000 N for a fixed rotation angle of 320°.

2) Fig 5(c) shows the variations of the displacements with the rotation angle in a revolution for an input load F = 1100N.

3) Fig 5(d) shows the variations of the displacements with the rotation angle in a revolution while the loads vary sinusoidally as 1100+400×sin(θ)(N).

From the simulation results obtained using the model of the RE mechanism and the experimental data, it can be seen that the precision of an actual revolute joint depends on geometric errors, elasticity of components and loads. Through the RE mechanism, a generic relationship among various measurable properties and loads for an actual revolute joint is established, The precision of an actual revolute joint and its load-dependent properties can be seen as the solution of the basic equations of a corresponding RE mechanism, this opens up a new application of the theory of mechanism in design of machine components.

6 Conclusions

A novel concept of an equivalent RE mechanism consisting of two spatial cams and 10 followers with springs is presented in the paper to simulate the effects of geometrical errors and elasticity of components in an actual revolute joint. The proposed mechanism is a new spatial mechanism with redundant degrees of freedom and elastic constraints.

The displacement of the RE mechanism can easily simulate precision and properties of an actual revolute joint by taking into account of component geometric errors and their flexibility. The numerical results for the example revolute joint correlate well with the measurements.

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