Spherical Parallel Mechanism with Variable Target Point

Tsuyoshi Ikeda, Yukio Takeda, and Daisuke Matsuura

Abstract This paper proposes a position-orientation decoupled parallel mechanism with five degrees of freedom, in which rotational motion of the output link around two axes is controlled by two inputs while translational motion of the target point, the center of rotation of the output link, is controlled by the other three inputs. This mechanism is composed of three connecting chains; one for controlling the position of the target point and two for generating rotational output motion. Conditions of kinematic structures of these chains are discussed and a concrete mechanism is shown. Inverse displacement analysis and Jacobian analysis of this mechanism are carried out to confirm its decoupled feature without encountering the singular point.

Key words: Robotics, Kinematics, Spherical Parallel Mechanism, Structural Synthesis, Position-Orientation Decoupled Mechanism, Displacement Analysis, Singularity.

1 Introduction

There are a lot of operations that require precise rotational output motion around two or three axes while the position of the rotation center being changed in a three dimensional space. As such examples, minimum invasive surgery under laparoscope and manufacturing of femoral head of prostheses done by robots are illustrated in Fig. 1. We consider a robot mechanism for such applications. In both cases, the actuators should be remotely located from the operation area in order that they would be protected from the working environment. From the safety point of view, the moving part of the mechanism should be as light as possible. Parallel mechanism is con-

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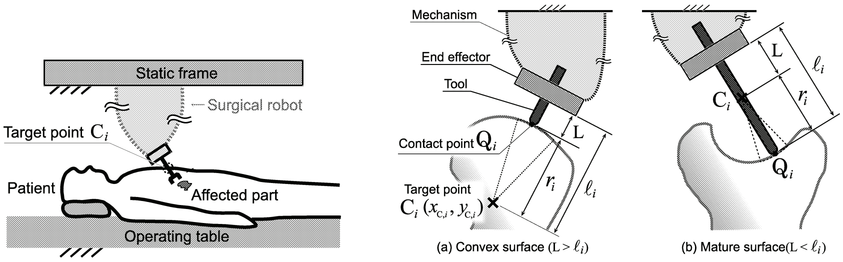


(a) minimum invasive surgery (b) manufacturing of prostheses

Fig. 1 Target applications

sidered to be one of the most appropriate candidates for such robot mechanisms, because all the actuators can be located on or close to the base and accurate motion can be achieved even under heavy load condition.

There are a lot of articles regarding kinematic analysis and synthesis of pure-rotational/spherical parallel mechanisms [1, 3, 12, 15]. However, there are a few articles [11, 20] in which design method or proposition of kinematic structure enabling variable position of the center of output rotation in such mechanisms have been discussed. Taking into consideration the simplicity of the control system, position-orientation decoupled parallel mechanism is considered to be one of the optimal choices. In literature, several position-orientation decoupled parallel mechanisms with six degrees of freedom (DOF) having symmetrical structure have been proposed [4, 5, 10, 13, 16, 18]. However, there are a few articles in which mechanisms with asymmetrical structure have been considered [2, 19]. Asymmetrical three-DOF rotational-translational parallel mechanisms [14] and parallel mechanisms generating three-DOF finite translation and two-DOF infinite rotation [9] have been figured out. In the present paper, we propose an asymmetrical position-orientation decoupled parallel mechanism with five DOF. In our previous work, an asymmetrical rotational parallel mechanism has been developed that can perform rotational output motion around two axes and fine translational motion to compensate for position error of the target point, the center of rotation of the output link. However, the actuators for changing the target point's position were located on the output link [11].

The present paper is organized as follows. In section 2, a basic structure of SPMVTP(Spherical Parallel Mechanism with Variable Target Point), one of the asymmetrical position-orientation decoupled parallel mechanisms, is proposed. In section 3, a kinematic structure of SPMVTP is clarified in which translational and rotational output motions are fully decoupled, and its inverse displacement analysis is discussed. In section 4, a numerical example is shown and the effectiveness of the mechanism is discussed.

2 Basic Structure of SPMVTP

2.1 Mechanism Configuration

Figure 2 shows the basic structure of SPMVTP which is composed of a target point controlling chain (TPC) and two rotational motion generating chains (RMC). In the figure, quadrangular prism and cylinder represent prismatic and revolute joints, respectively. A circle represents a joint with a single DOF of arbitrary type. Types of these joints represented by circles and their axis directions determine the kinematic structure of SPMVTP. The unit vector $\mathbf{w}_{j,i}$ gives the direction either of the rotational motion if $\mathrm{J}_{j,i}$ is a revolute joint or of the translational motion if $\mathrm{J}_{j,i}$ is a prismatic joint, where $\mathrm{J}_{j,i}$ denotes the j-th joint of the i-th chain.

TPC is composed of two passive revolute joints and a translational mechanism with three DOF. In the figure, a serial chain with three active prismatic joints ($\mathrm{J}_{1,1} \sim \mathrm{J}_{3,1}$) is shown as the translational mechanism. The two revolute joints meet at a point P, which is the center of the rotation of the output link, called hereafter "target point". The translational mechanism determines the position of the target point.

RMC is a serial chain with five passive joints ($\mathrm{J}_{2,i} \sim \mathrm{J}_{6,i}; i = 2,3$) and an active joint ($\mathrm{J}_{1,i}$). Rotational motion around an axis and full translational motion of the output link is constrained by fixing all active joints in TPC. To completely constrain the motion of the output link when all active joints of RMC as well as TPC are fixed, a rotational motion around an axis should be constrained by fixing the active joint of each RMC. In addition to this, the total three constraints with respect to the rotational motion should be linearly independent in order to avoid singularity. Kinematic structures satisfying these conditions are candidates for SPMVTP, in which an output motion composed of three-DOF translational motion and two-DOF rotational motion can be controlled by the five active joints.

2.2 Velocity Relationship

For SPMVTP which satisfies with the conditions mentioned in the previous subsection, relationship between the input velocity and output velocity is derived. Input velocity is denoted as $[\dot{\theta}^{\mathrm{T}}\ \dot{\mathbf{q}}^{\mathrm{T}}]$ ($\dot{\theta} = [\dot{\theta}_{1,2} \quad \dot{\theta}_{1,3} \quad 0]^{\mathrm{T}}, \dot{\mathbf{q}} = [\dot{q}_{1,1} \quad \dot{q}_{2,1} \quad \dot{q}_{3,1}]^{\mathrm{T}}$) where the velocity of $\mathrm{J}_{j,i}$ is denoted by $\dot{\theta}_{j,i}$ (revolute joint) or by $\dot{q}_{j,i}$ (prismatic joint) of RMCs (i=2,3) and of TPC (i=1), respectively. Output velocity is denoted as $[\omega_{\mathrm{P}}^{\mathrm{T}} \mathbf{v}_{\mathrm{P}}^{\mathrm{T}}]^{\mathrm{T}}$ where ω_{P} and $\mathbf{v}_{\mathrm{P}}$ represent the angular velocity and the velocity at P of the output link, respectively. Relationship between the input velocity and output velocity is written as

$$J_{\mathrm{T}} \begin{bmatrix} \omega_{\mathrm{P}} \\ \mathbf{v}_{\mathrm{P}} \end{bmatrix} = \begin{bmatrix} J_{\mathrm{A}} & J_{\mathrm{B}} \\ 0 & J_{\mathrm{C}} \end{bmatrix} \begin{bmatrix} \omega_{\mathrm{P}} \\ \mathbf{v}_{\mathrm{P}} \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ \dot{\mathbf{q}} \end{bmatrix} \quad (1)$$

where

$$J_{\mathrm{A}} = \begin{bmatrix} \frac{\hat{\mathbf{m}}_2^{\mathrm{T}}}{\hat{\mathbf{m}}_2^{\mathrm{T}}\hat{\omega}_{1,2}+\hat{\mathbf{f}}_2^{\mathrm{T}}\hat{\mathbf{v}}_{1,2}} \\ \frac{\hat{\mathbf{m}}_3^{\mathrm{T}}}{\hat{\mathbf{m}}_3^{\mathrm{T}}\hat{\omega}_{1,3}+\hat{\mathbf{f}}_3^{\mathrm{T}}\hat{\mathbf{v}}_{1,3}} \\ (\mathbf{w}_{4,1}\times\mathbf{w}_{5,1})^{\mathrm{T}} \end{bmatrix}, J_{\mathrm{B}} = \begin{bmatrix} \frac{\hat{\mathbf{f}}_2^{\mathrm{T}}}{\hat{\mathbf{m}}_2^{\mathrm{T}}\hat{\omega}_{1,2}+\hat{\mathbf{f}}_2^{\mathrm{T}}\hat{\mathbf{v}}_{1,2}} \\ \frac{\hat{\mathbf{f}}_3^{\mathrm{T}}}{\hat{\mathbf{m}}_3^{\mathrm{T}}\hat{\omega}_{1,3}+\hat{\mathbf{f}}_3^{\mathrm{T}}\hat{\mathbf{v}}_{1,3}} \\ \mathbf{0}_3 \end{bmatrix}, J_{\mathrm{C}} = \begin{bmatrix} \mathbf{w}_{1,1}^{\mathrm{T}} \\ \mathbf{w}_{2,1}^{\mathrm{T}} \\ \mathbf{w}_{3,1}^{\mathrm{T}} \end{bmatrix}, \quad (2)$$

respectively. $\mathbf{S}_{1,i} = [\hat{\omega}_{1,i}^{\mathrm{T}} \hat{\mathbf{v}}_{1,i}^{\mathrm{T}}]^{\mathrm{T}} (i = 2,3)$ and $\mathbf{S}_{\mathrm{RA},i} = [\hat{\mathbf{f}}_i^{\mathrm{T}} \hat{\mathbf{m}}_i^{\mathrm{T}}]^{\mathrm{T}} (i = 2,3)$ represent the joint screw of the active joint and the constraint wrench imposed by fixing the active joint of RMC, respectively.

Equations for forward velocity calculation is obtained as

$$\left.\begin{array}{l} \omega_{\mathrm{P}} = J_{\mathrm{A}}^{-1}(\dot{\theta} - J_{\mathrm{B}} J_{\mathrm{C}}^{-1}\dot{\mathbf{q}}) \\ \mathbf{v}_{\mathrm{P}} = J_{\mathrm{C}}^{-1}\dot{\mathbf{q}} \end{array}\right\}. \quad (3)$$

It is known from the equation that input motion of RMC generates pure rotational motion while input motion of TPC generates rotational motion coupled with translational motion of the output link. This means that rotational output motion is decoupled from translational motion in SPMVTP regardless of the kinematic structure of RMC.

3 Strucure of Fully Decoupled SPMVTP

3.1 Kinematic Structure of RMC

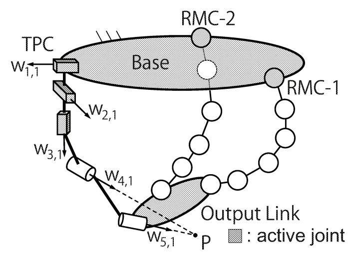
Let us consider a case of $J_{\mathrm{B}} = 0$ in Eq. (1). In such a case, the velocity equation becomes

$$\left.\begin{array}{l} \omega_{\mathrm{P}} = J_{\mathrm{A}}^{-1}\dot{\theta} \\ \mathbf{v}_{\mathrm{P}} = J_{\mathrm{C}}^{-1}\dot{\mathbf{q}} \end{array}\right\}. \quad (4)$$

Mechanisms satisfying this condition are called *"fully decoupled SPMVTP"*. In this section, its kinematic structure is investigated. Singularity of fully decoupled SPMVTP can be investigated by the determinants of sub-matrices J_{A} and J_{C}. If the kinematic structure of TPC shown in Fig. 2 is employed, $\det J_{\mathrm{C}}$ is always 1. In what follows, the singularity defined as $\det J_{\mathrm{A}} = 0$ will be considered.

From Eq. (1), constraint screw of RMC with its active joint fixed for fully decoupled SPMVTP should be in the form of $\mathbf{S}_{\mathrm{RA},i} = [\mathbf{0}^{\mathrm{T}} \ \hat{\mathbf{m}}_i^{\mathrm{T}}]^{\mathrm{T}} (i = 2,3)$. This means that thanks to the constraint imposed by the two RMCs with their inputs being fixed, the orientation of the output link does not change by the input motion of TPC for changing the position of target point.

Starting from the kinematic structures for translational parallel mechanism (TPM) with three serial connecting chains, we figured out kinematic structures of RMC for fully decoupled SPMVTP. The structural conditions for the connecting chain of TPM are summarized in [7, 8, 17]. Kinematic structures of RMC for fully decoupled SPMVTP can be obtained by adding a revolute joint at the base to the kinematic



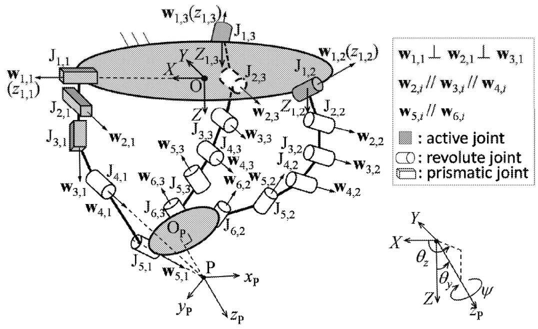


Fig. 2 Basic composition of SPMVTP. **Fig. 3** An example of fully decoupled SPMVTP.

chains for TPM so that the conditions for connecting chain of TPM are satisfied even when the added revolute joint is arbitrarily positioned.

An example of fully decoupled SPMVTP is shown in Fig. 3 in which six revolute joints are used as RMC. Kinematic structure and dimensions of RMC should be determined so that the following conditions are satisfied.

1. The axes of the revolute joints $\mathbf{w}_{2,i}$ to $\mathbf{w}_{4,i}$ should be parallel.
2. The axes of the revolute joints $\mathbf{w}_{5,i}$ and $\mathbf{w}_{6,i}$ should be parallel while the axes of joints $\mathbf{w}_{4,i}$ and $\mathbf{w}_{5,i}$ should not be parallel.
3. The axes of the revolute joints $\mathbf{w}_{1,i}$ and $\mathbf{w}_{2,i}$ should not be parallel to avoid architectural singularity of RMC [6]
4. Rank of the Jacobian matrix with respect to RMC as a serial chain should be 6.
5. Rank of the sub-matrix J_{A} should be 3.

The orientation angles $(\theta_y, \theta_z, \psi)$ shown in Fig. 3 are used to represent the orientation of the output link. The kinematic constants are defined as shown in Fig. 4. Other kinematic constants are defined using DH parameters (a, d, α and θ are link length, offset length, twist angle, and rotation angle, respectively).

3.2 Inverse Displacement Analysis of SPMVTP

Pose (position of P and orientation) of the output link is represented by a 4×4 transformation matrix T_{P} as

$$T_{\mathrm{P}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \mathbf{p} & & R & \end{bmatrix} \tag{5}$$

where $\mathbf{p}$ and $R = [\mathbf{e}_x \ \mathbf{e}_y \ \mathbf{e}_z]^{\mathrm{T}}$ are the position vector of P and the 3×3 rotation matrix representing the orientation, respectively. Transformation matrix can be written as a function of kinematic constants and joint variables of each chain. Those with respect to TPC and RMCs are denoted as $T_{\mathrm{P,TPC}}$ and $T_{\mathrm{P,RMC},i}$, respectively. Inverse displacement analysis of SPMVTP is defined as a problem to solve the following equation

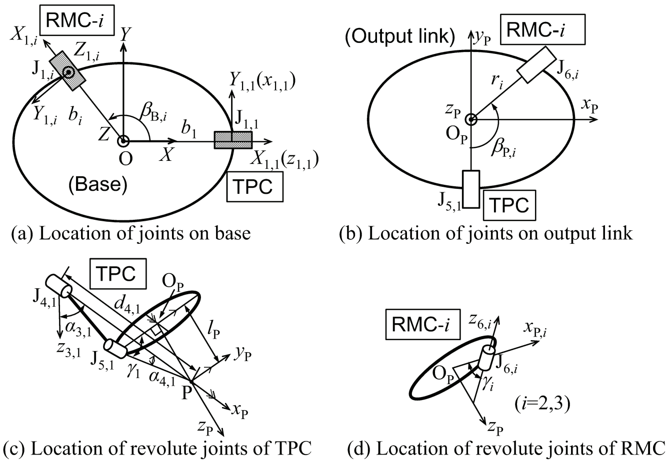


Fig. 4 Definition of kinematic constants

with respect to $\theta_{1,i}$ and $q_{j,1}(i=2,3; j=1,2,3)$ for a given pose $(X_P, Y_P, Z_P, \theta_y, \theta_z)$.

$$T_P = T_{P,TPC} = T_{P,RMC,i}(i=2,3) \tag{6}$$

However, since SPMVTP is a mechanism with five DOF, T_P is not fully defined by the given output pose. Then, the following procedure for the inverse displacement analysis of SPMVTP has been developed.

1. Since $\mathbf{e}_z$ is a function of θ_y and θ_z, passive joint displacements of TPC $\theta_{4,1}$ and $\theta_{5,1}$ can be obtained by solving $\mathbf{e}_z = \mathbf{e}_{z,TPC}$. There are two solutions.
2. Input displacements $q_{j,1}(j=1,2,3)$ can be uniquely obtained from

$$\mathbf{p} = \sum_{j=1}^{3} q_{j,1}\mathbf{w}_{j,1}. \tag{7}$$

3. T_P can be fully determined by $T_{P,TPC} \to T_P$ using the results of steps 1 and 2.
4. Solutions of $\theta_{1,i}(i=2,3)$ can be obtained by solving $T_P = T_{P,RMC,i}$. There are eight real solutions at maximum for each RMC.

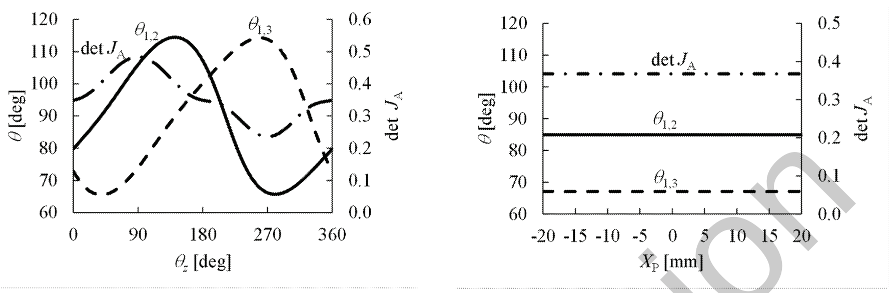
4 Numerical Example

For a mechanism with kinematic constants shown in Table 1, inverse displacement analysis has been done following the procedure described in the previous section. Results for the following two cases of the output motion are shown in Fig. 5.

1. $(X_P, Y_P, Z_P) = (0, 0, 130)$[mm], $\theta_y = 30^\circ$, $\theta_z = [0:360]^\circ$

Table 1 Kinematic constants of mechanism

Symbols	Values	Symbols	Values	Symbols	Values
l_P	6.84 mm	$\beta_{B,3}$	135°	$d_{1,i}$	50 mm
b_1	60 mm	$\beta_{P,2}$	45°	$a_{2,i}$	50 mm
$\alpha_{3,1}$	90°	$\beta_{P,3}$	225°	$a_{3,i}$	60 mm
$d_{4,1}$	60 mm	b_i	110 mm	$a_{4,i}$	10 mm
$\alpha_{4,1}$	90°	$\alpha_{0,i}$	45°	$a_{5,i}$	40 mm
γ_1	20°	$\theta_{0,i}$	270°	γ_i	30°
$\beta_{B,2}$	135°	$a_{1,i}$	0 mm	r_i	35 mm



(a) case-1 (b) case-2

Fig. 5 Input displacement of RMC and det J_A

2. $X_P = [-20:20]$[mm], $(Y_P, Z_P) = (0, 130)$[mm], $\theta_y = 30°$, $\theta_z = 0$

In these figures, determinant of the sub-matrix J_A is also shown. From the figures, it is known that the proposed mechanism can achieve a fully-decoupled output motion without encountering singularity.

5 Conclusions

In the present paper, an asymmetrical five-DOF fully-decoupled parallel mechanism has been proposed and its kinematic study has been carried out. Our conclusions are summarized as follows.

1. A kinematic structure of spherical parallel mechanism with variable target point, which is composed of a target point controlling chain and two rotational motion generating chains, has been figured out.
2. A procedure for inverse displacement analysis of the mechanism has been clarified.
3. Effectiveness of the mechanism has been confirmed through a numerical example of inverse displacement analysis with a check of singularity.

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