An improved Force Distribution Algorithm for Over-constrained Cable-driven Parallel Robots

Andreas Pott

Abstract In this paper we present an improved method to compute force distributions for cable-driven parallel robots. We modify the closed-from solutio such that the region where a solution is found is extended almost to the theoretical maximum, i.e. the wrench-feasible workspace. At the same time continuity along trajectories as well as real-time efficiency are maintained. The algorithm's complexity and thus the computational burden scales linearly in the number of redundant cables. Therefore, the algorithm can also be used for highly redundant cable robots. The proposed algorithm is compared to known methods and computational results are presented based on the IPAnema prototype.

Key words: cable-driven parallel robots, force distribution, closed-form, real-time

1 Introduction

Cable-driven parallel robots are a special kind of parallel manipulators where the rigid struts are replaced by flexible elements. Many cable robots facilitate more cables m than degrees-of-freedom n in order to withstand applied wrenches $\mathbf{w}$ in arbitrary directions. Therefore, these robots are redundantly actuated and static or dynamic balancing of the robots requires a distribution of actuator forces amongst the cables. The force and torque equilibrium for cable robots is usually written in matrix form as follows [13]

$$\mathbf{A}^{\mathrm{T}}\mathbf{f} + \mathbf{w} = \mathbf{0} \quad \text{with} \quad 0 < f_{\min} \leq f_i \leq f_{\max}, \quad i \in [1, m], \tag{1}$$

where the matrix $\mathbf{A}^{\mathrm{T}}$ is the pose dependent structure matrix or sometimes also called wrench matrix, $\mathbf{f}$ are the forces in the cables, and $f_{\min}$, $f_{\max}$ represent the minimum and

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maximum feasible cable forces, respectively. The presented structure equations also covers the dynamic case when using d'Alembert's principle to add the inertial forces to the applied wrench **w**. Computing force distributions for the cables e.g. for control requires finding solutions to the structure equations above, where the linear system is under-determined for the cable forces **f**. Therefore, infinitely many solutions are consistent with the structure equations but these solutions are not necessarily in the feasible region given by the bounds $f_{\min}, f_{\max}$. Given that such solution exists the problem addressed in this paper is to efficently compute one solution that is continuous along a trajectory of the robot's mobile platform.

Different approaches were proposed in the literature to calculate force distributions, and each approach delivers force distributions with different characteristics while requiring varying computational efforts:

- Gradient-based optimization using a p-norm for $r > 1$ (Verhoeven's method) [13]
- Specialized optimization for p-norm with $p = 4$ [5]
- Constrainted l_1-norm optimization [12]
- Minimizing p-norm with Dykstra method [6]
- Closed-form solution for $p = 2$ [11]
- Linear programming [1, 10]
- Quadratic programming for $r = 2$ [4] and for $r = 0$ [8][1]
- Nonlinear programming [3]
- Barycentric approach [9] and improved implementations [7]
- Kernel method [13, p. 58] for $r = 1$
- Weighted sum of solution space vertices [3]
- Available wrench set [2]

A comparison of some force distribution methods and their properties is given in Tab. 1. We briefly explain the properties listed in the head of the table. An algorithm is said to be *real-time capable* if the computation time is reasonably short, the worst-case computation time can be strictly bounded, and a real-time implementation was reported in the literature. Some iterative methods were successfully used for computation in real-time although their worst-case computational time was not determined. The *force niveau* may be choosen, e.g. the algorithm may aim at finding minimal (lo), maximal (hi), average (mi), or any solution (any). Furthermore, there might be a parameter (param) that allows to smoothly adjust the niveau of tension between low and high. A couple of authors [13, 11] reported approaches that may fail to find force distributions for special poses of the wrench-feasible workspace. Full *workspace coverage* indicates that for every pose of the wrench-feasible workspace a solution can be found. For some methods it is not known (n.a.) if they cover the full workspace. An algorithm is said to provide *continuity*, if continuous trajectories in the pose $\mathbf{r}, \mathbf{R}$ as well as in the applied wrench **w** produce continuous trajectories in the cable forces **f**, except for crossing a singularity. Some methods are limited to a certain degree-of-redundancy $r \leq 0$ either because they are specific or because their implementation can hardly be generalized to arbitrary r. The evaluation of

[1] Li [8] only deals with the non redundant case $r = 0$, i.e. six cables and six degrees-of-freedom.

Table 1 Comparison of the different methods to compute force distributions.

method	real-time capable	force niveau	workspace coverage	continuity	max. redun- dancy	compu- tational speed
linear programming	no	any	yes	no	any	fast
quadratic programming	yes	hi,lo	n.a.	yes	any	medium
gradient-based optimization	no	param	no	yes	any	medium
Dykstra	no	any	yes	no	any	slow
closed-form	yes	any	no	yes	any	fast
barycentric	yes	mi	yes	yes	$r = 2$	fast
weighted sum	yes	mi	yes	mostly	any	medium
kernel method	yes	hi, mi, lo	yes	yes	$r = 1$	fast
available wrench set	no	hi, mi, lo	yes	no	any	slow

the computational speed is problematic because it requires comparable implementations which are not available for all methods reported in the literature. Anyway, it was tried to set up a basic ranking taking into account how complex the underlying numerical method is. For example, linear system solving is considered to be faster than inverting a matrix, which in turn is faster than computing a singular value decomposition. Designing a real-time system might become involved if an advanced numerical algorithm such as advanced optimization or singular value decomposition shall be used. This is due to lack of appropriate real-time capable implementations of the algorithm although the algorithm is part of every state-of-the-art numerical toolbox. The computational speed depends on the degree-of-redundancy in addition to the algorithm's complexity. For this assessment, a low degree-of-redundancy was assumed.

From the table it can be seen that no method is known that is real-time capable, covers the full workspace, delivers continuous solution for control, and works for robot with arbitrary degree-of-redundancy. In this paper, we propose an improved variant of the closed-form method to overcome its shortcomings with respect to workspace coverage while maintaining an acceptable computation time for usage in a real-time controller.

2 Improved closed-form method

Lately, we developed a formula to compute a solution for the force distribution problem in closed-form [11]. The basic idea of the method is to perform a coordinate transformation to the medium feasible cable force $\mathbf{f}_m = \frac{1}{2}(\mathbf{f}_{\min} + \mathbf{f}_{\max})$. This also changes parts of the optimization problem from constrained optimization to pure minimization. The cable forces $\mathbf{f}$ can be computed as [11]

$$\mathbf{f} = \mathbf{f}_m + \mathbf{f}_v = \mathbf{f}_m - \mathbf{A}^{+\mathrm{T}}(\mathbf{w} + \mathbf{A}^{\mathrm{T}}\mathbf{f}_m), \tag{2}$$

where $\mathbf{A}^{\mathrm{T}}$ and $\mathbf{A}^{+\mathrm{T}}$ are the structure matrix and its pseudo-inverse, respectively, and $\mathbf{w}$ is the applied wrench. As discussed in [11] this formula might fail to provide a feasible solution although such a solution exists, if the magnitude of the variable part $\mathbf{f}_v$ of the force distribution is in the range

$$\frac{1}{2} f_m \leq |\mathbf{f}_v|_2 \leq \frac{1}{2}\sqrt{m} f_m \tag{3}$$

If $|\mathbf{f}_v|_2$ violate the upper limit no solution exists and if it is below the lower limit the distribution is feasible. This undefined case occurs amongst others close to the boarder of the wrench-feasible workspace, for robots with many redundant cables, and for redundant robots in suspended configuration.

In the following, we propose to extend the method such that feasible force distributions are found in almost all cases where the original method fails.[2] The closed-form solution is guaranteed to fulfill the force equilibrium but may violate the force limits. Thus, the following approach is proposed:

1. Eq. (2) is used to compute an estimate for the force distribution. If this initial guess already fulfills the cable force conditions we have the sought solution and stop the algorithm.
2. Otherwise, let i be the cable with the largest force over (under) the maximum (minimum) feasible cable force. If one moves from this distribution along the spanning base of the structure matrix kernel one must cross the value where f_i reaches its maximum (minimum) feasible value.
3. Therefore, it is assumed[3] that a feasible force distribution minimizing the 2-norm can only be found if this cable force is fixed to its maximum (minimum) value $f_{\max}$ ($f_{\min}$). Using a constant value for cable force f_i simplifies the force distribution problem as follows

$$\mathbf{A}'^{\mathrm{T}}\mathbf{f}' + \mathbf{w}' = \mathbf{0} \quad \text{with} \quad \mathbf{w}' = f_{\max}\left[\mathbf{A}^{\mathrm{T}}\right]_i + \mathbf{w}, \qquad \left(\mathbf{w}' = f_{\min}\left[\mathbf{A}^{\mathrm{T}}\right]_i + \mathbf{w}\right), \tag{4}$$

   where $\mathbf{A}'^{\mathrm{T}}$ and $\mathbf{f}'$ is the structure matrix and the cable forces vector with the i-th column/element dropped, respectively. $[\mathbf{A}^{\mathrm{T}}]_i$ denotes the i-th column of the matrix $\mathbf{A}^{\mathrm{T}}$. Thus, we have reduced the actuator redundancy r by one.
4. Now we compute the solution by recursively reducing the order and computing the closed-form solution by going to step 1 until:
   a. we find a feasible distribution,
   b. the remaining degree-of-redundancy is negative $r < 0$, then no solution exists,
   c. Eq. (3) proofs that no feasible solution exists because because the computed force violates the right part of Eq. (3)

Therefore, we find the desired cable force distribution (if it exists) with at most r evaluations of the closed-form formula (2).

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[2] In numerical studies some poses were found on the very boarder of the workspace where the presented methods fails to find a solution.

[3] Unfortunately, we have no formal proof that this holds true in general.

Table 2 IPAnema 1 nominal geometric parameters: platform vectors $\mathbf{b}$ and base vectors $\mathbf{a}$

cable i	base vector $\mathbf{a}_i$	platform vector $\mathbf{b}_i$
1	$[-2.0, 1.5, 2.0]^{\mathrm{T}}$	$[-0.06, 0.06, 0.0]^{\mathrm{T}}$
2	$[2.0, 1.5, 2.0]^{\mathrm{T}}$	$[0.06, 0.06, 0.0]^{\mathrm{T}}$
3	$[2.0, -1.5, 2.0]^{\mathrm{T}}$	$[0.06, -0.06, 0.0]^{\mathrm{T}}$
4	$[-2.0, -1.5, 2.0]^{\mathrm{T}}$	$[-0.06, -0.06, 0.0]^{\mathrm{T}}$
5	$[-2.0, 1.5, 0.0]^{\mathrm{T}}$	$[-0.06, 0.06, 0.0]^{\mathrm{T}}$
6	$[2.0, 1.5, 0.0]^{\mathrm{T}}$	$[0.06, 0.06, 0.0]^{\mathrm{T}}$
7	$[2.0, -1.5, 0.0]^{\mathrm{T}}$	$[0.06, -0.06, 0.0]^{\mathrm{T}}$
8	$[-2.0, -1.5, 0.0]^{\mathrm{T}}$	$[-0.06, -0.06, 0.0]^{\mathrm{T}}$

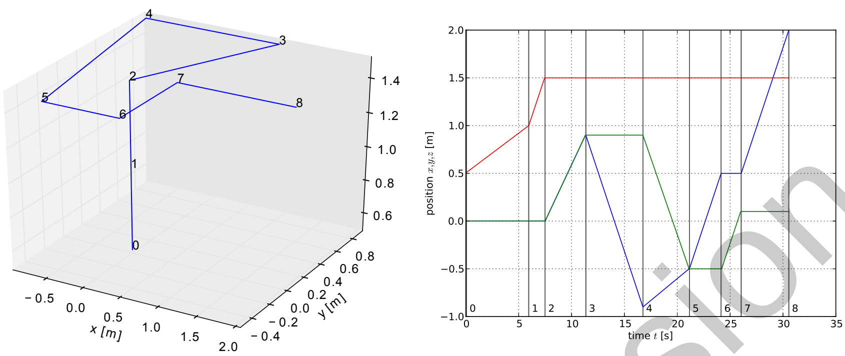
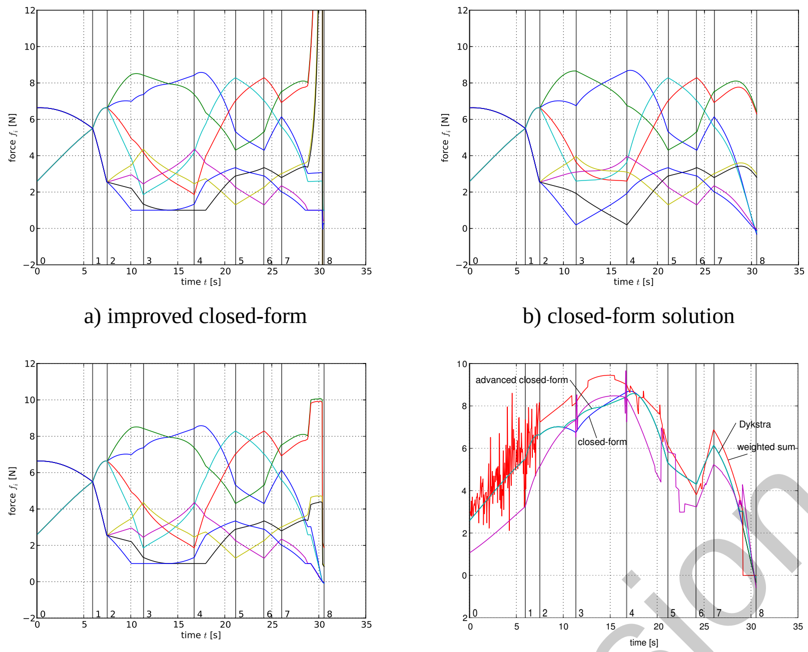


Fig. 1 Test trajectory used for the evaluation

3 Simulation results and computation time

For the numerical examples we use the geometrical parameters of the cable robot IPAnema 1 given in Tab. 2. To compare different algorithms for force distribution a sample trajectory is used which is depicted in Fig. 1. The waypoints of the trajectory are indicated by number 0 to 8 and the following plots with cable forces against time have additional marks above the x-axis indicating the waypoints for better reference. Position was linearly interpolated between the waypoints and the trajectory was choosen such that the robot moves in different regions of the workspace and finally crosses the boarder of the wrench-feasible workspace between waypoint 7 and 8. The force limits were $f_{\mathrm{min}} = 1$ and $f_{\mathrm{max}} = 10\,\mathrm{N}$. Inertia effects of the platform were neglected.

Fig. 2a illustrates the proposed improved algorithms based on closed-form estimation and correction for the remaining cables. From the diagram one can see that the force distribution is continuous along the trajectory and the magnitude of the forces are on a medium level. When approaching the boarder of the workspace (e.g. between $t = 10.0\,\mathrm{s}$ and $t = 17.0\,\mathrm{s}$ one or two cable forces reach the minimum cable force and remain constant at the limit. It can be seen from the shape of the diagram that the cable forces quickly increase after leaving the workspace. Anyway, the force distributions remain continuous after crossing the boarder of the workspace.



a) improved closed-form   b) closed-form solution

c) Dykstra method   d) comparison of different methods

Fig. 2 Comparison of different methods to compute the force distribution.

In Fig. 2b the force distribution is shown for the original closed-form method for comparison. When the platform remains in the inner region of the workspace the results match the force distributions computed with the correction technique. Close to the boarder of the workspace the closed-form formula fail to compute force distributions although such distributions exist as it can be seen between waypoint 2 and 5 and also between 7 and 8, where the closed-form solution is not able to prevent some cables from violating the lower force limits.

Cable forces computed with the Dykstra methods are presented in Fig. 2c. It can be observed for the Dykstra method that some cable forces get limited to the minimal values when the boarder of the workspace is approached. After crossing the workspace board between waypoint 7 and 8 force distributions computed with Dykstra show a different behaviour compared to the proposed scheme.

In Fig. 2d the computed forces for the first cable f_1 is compared for different methods. Some methods like the uncorrected weighted sum method do not even provide continuous shapes for the forces which becomes evident between waypoint 0 and 2. Other methods show discrete steps at certain points on the trajectory.

A comparison of the computation time is difficult because the computation time is influenced by the maturity of the implementation as well as the underlying numerical algorithms, the used compiler, the CPU of the real-time system, and the

Table 3 Comparison of computation time on an Intel Core i5-3320M 2.6 GHz,Visual C++ 2010

algorithm	*calculation time [ms]*	*relative time*	*evaluations per ms*
closed-form	1173	100%	293
advanced closed-form	3359	286%	102
Dykstra	71612	6103%	5
weighted sum	48512	4134%	7

operating system. In the performance test presented here, four algorithms were used for workspace computation with around 344 000 evaluations on an Intel Core i5-3320M. As an estimate some numbers are given in Tab. 3. The table lists both absolute and relative computation time to allow for comparison amongst the algorithms as well as to present an estimate on the usability in a real-time controller. As expected the closed-form solution works faster than its improved version but the difference is comparibly small. The performance advantages of the presented method over the iterative Dykstra method and over the exhausive search of weighted-sum method can be explained by the more efficient search strategy. Each iteration step of the advanved close-form method is used to fix at least one component in force vector. Both closed-form methods allow for many evaluations based on a controller cycle time of 1 ms and their implementations only requires matrix multiplication and solving of a linear system.

4 Conclusions

In this paper we proposed an improved algorithm to compute force distributions for over-constrained cable-driven parallel robot under real-time requirements. The improved version overcomes a major drawback of the closed-form solution, i.e. that the algorithm failed to find force distributions especially close to the boarder of the workspace. The improved algorithm is still applicable for robots with a large number of cables. Although the computational time of the presented algorithm is now linear in the number of redundant cables it still provides a solution for highly redundant cable robots in reasonable time.

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