On the Minimum 2-Norm Positive Tension for Wire-Actuated Parallel Manipulators

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Abstract Utilizing the Moore-Penrose generalized inverse of the Jacobian matrix of wire-actuated parallel robot manipulators, one or more wire tensions could be negative. In this paper, a methodology for calculating positive wire tensions, with minimum 2-norm for tension vector, is presented. A planar parallel manipulator is simulated to illustrate the proposed methodology.

Key words: wire-actuated robots, positive wire tension, minimum 2-norm solution.

1 Introduction

In wire/cable-actuated parallel robot manipulators, also known as wire/cable-driven parallel manipulators, the motion of mobile platform (end effector) is constrained by wires/cables. Because wires act in tension and cannot exert forces in both directions along their lines of action, i.e., their inputs are unidirectional and irreversible, to fully constrain an m degrees of freedom (DOF) rigid body suspended by wires, in the absence of gravity and external force/moment (wrench), the number of wires should be larger than the DOF of manipulator (Figures 1), i.e., $n \geq m+1$.

The manipulator failure could be defined as any event that affects its performance such that the manipulator cannot complete its task as required. Wire-actuated manipulators could fail because of the hardware and/or software failures, including failure of a wire, sensor, actuator, or transmission mechanism; as well as computational failure. These failures could result in the loss of DOF, actuation, motion constraint, and information (Notash and Huang, 2003). From the force point of view, the failure of manipulator occurs if the wire does not provide the required force/torque, e.g., when the actuator force/torque is lost partially or fully or the actuator is saturated. This could also happen when the wire is broken or slack (zero tension), wire

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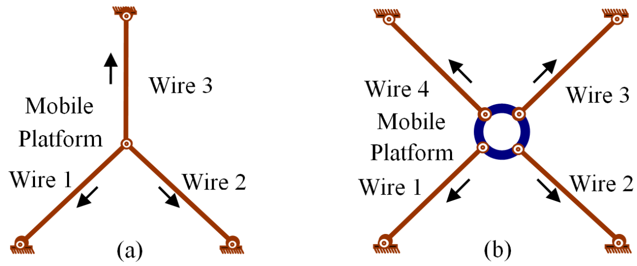

Fig. 1 Planar wire-actuated manipulators (a) 2 DOF; and (b) 3 DOF.

is jammed (constant length), or its actuating mechanism malfunctions such that a different wire force is provided.

For a given wrench to be applied/resisted by the mobile platform of wire-actuated parallel manipulators, because $n \geq m+1$, there are infinite solutions for the wire tension vector. The minimum 2-norm solution could result in negative tension for wires, which is not acceptable, and generally the homogeneous solution is used to adjust the tension to positive values if the platform position and orientation (pose) is within the wrench closure workspace, e.g., (Roberts et al., 1998). Due to space limitation, a review of pertinent literature is not included.

In this paper, formulation of a non-negative wire tension vector for wire-actuated parallel manipulators is investigated when the DOF of manipulator m is one less than the number of wires n, i.e., $n = m+1$. In Section 2, the implementation of the methodology of (Notash, 2012a) for achieving minimum 2-norm positive wire tension vector is presented. Simulation results are reported in Section 3. The article concludes with Section 4.

2 Wrench Recovery for Negative Wire Tension

For the n-wire-actuated parallel manipulators, the $n \times 1$ vector of wire forces $\boldsymbol{\tau} = \left[\tau_1 \cdots \tau_n\right]^T$ is related to the $m \times 1$ vector of forces and moments (wrench) $\mathbf{F}$ applied by the platform with the $m \times n$ transposed Jacobian matrix $\mathbf{J}^T$ as

$$\mathbf{F} = \mathbf{J}^T\boldsymbol{\tau} = \left[\mathbf{J}_1^T\ \mathbf{J}_2^T\ \cdots\ \mathbf{J}_i^T\ \cdots\ \mathbf{J}_{n-1}^T\ \mathbf{J}_n^T\right]\boldsymbol{\tau} = \sum_{j=1}^{n}\mathbf{J}_j^T\tau_j \tag{1}$$

where $m \leq 6$ depending on the dimension of task space. Column j of $\mathbf{J}^T$, $\mathbf{J}_j^T$, is a zero pitch screw corresponding to the wrench applied on the platform by the jth wire/actuator. The solution of $\mathbf{F} = \mathbf{J}^T\boldsymbol{\tau}$ for the wire tensions is

$$\boldsymbol{\tau} = \boldsymbol{\tau}_p + \boldsymbol{\tau}_h = \mathbf{J}^{\#T}\mathbf{F} + \left(\mathbf{I} - \mathbf{J}^{\#T}\mathbf{J}^T\right)\ \mathbf{k} = \mathbf{J}^{\#T}\mathbf{F} + \mathbf{N}\boldsymbol{\lambda} \tag{2}$$

where $\mathbf{J}^{\#T}$ is the Moore-Penrose generalized inverse of $\mathbf{J}^T$, $\boldsymbol{\tau}_p = \mathbf{J}^{\#T}\mathbf{F}$ is the minimum 2-norm (particular) solution, and $\boldsymbol{\tau}_h = \left(\mathbf{I} - \mathbf{J}^{\#T}\mathbf{J}^T\right)\mathbf{k}$ and $\boldsymbol{\tau}_h = \mathbf{N}\boldsymbol{\lambda}$ represent the homogeneous solution. $\left(\mathbf{I} - \mathbf{J}^{\#T}\mathbf{J}^T\right)\mathbf{k}$ is the projection of an $n \times 1$ vector $\mathbf{k}$ onto the null space of $\mathbf{J}^T$. Columns of the $n \times (n-m)$ matrix $\mathbf{N}$ correspond to the orthonormal basis of the null space of $\mathbf{J}^T$, referred here as the null space vectors, and $\boldsymbol{\lambda}$ is an $(n-m)$-vector. When one or more entries of $\boldsymbol{\tau}_p = \mathbf{J}^{\#T}\mathbf{F}$ are negative the wire tensions could be adjusted by identifying the correctional tension $\boldsymbol{\tau}_h$ that would set all the wire tensions to positive values provided the manipulator pose is in the wrench closure workspace. The adjusted wire tensions should satisfy the tension limits $\tau_{\min} \leq \tau_{p\,l} + \tau_{h\,l} = \tau_{p\,l} + n_l\lambda \leq \tau_{\max}$, for $l = 1, \cdots, n$, where the entries of $\boldsymbol{\tau}_h$ corresponding to negative τ_p must be positive in order to have non-negative tension vector after adjustment.

2.1 Conditions for Non-Negative Wire Tension

When the platform pose is in the wrench closure workspace of manipulator a non-negative solution to $\boldsymbol{\tau} = \boldsymbol{\tau}_p + \boldsymbol{\tau}_h = \mathbf{J}^{\#T}\mathbf{F} + \mathbf{N}\boldsymbol{\lambda}$ exists. The criteria for non-negative wire tension could be defined based on the orthonormal basis of the null space of the $m \times n$ transposed Jacobian matrix $\mathbf{J}^T$ and of the $m \times (n+t)$ augmented transposed Jacobian matrix $\mathbf{J}_{aug}^T$ of $\mathbf{J}_{aug}^T\boldsymbol{\tau}_{aug} = \left[\mathbf{J}_1^T\ \mathbf{J}_2^T\ \cdots\ \mathbf{J}_{n-1}^T\ \mathbf{J}_n^T\ \mathbf{W}_1^T \cdots \mathbf{W}_t^T\right]\boldsymbol{\tau}_{aug} = \mathbf{0}$. $\mathbf{J}_{aug}^T$ and $\boldsymbol{\tau}_{aug}$ are formed by re-writing equation (1) as $\mathbf{J}^T\boldsymbol{\tau} - \mathbf{F} = \mathbf{0}$ and augmenting $\mathbf{J}^T$ and $\boldsymbol{\tau}$ respectively with t wrenches and t number of 1's corresponding to t non-zero components of $\mathbf{F}$, where $t \leq m \leq 6$. $\mathbf{W}^T$ is a wrench with zeros for all entries except for the one that is equal to the negative of corresponding non-zero entry of $\mathbf{F}$.

The orthonormal basis of the null space of the $m \times n$ transposed Jacobian matrix $\mathbf{J}^T$, with full-row rank, is defined by $n-m$ number of n-vectors, i.e., the dimension of the null space vectors of $\mathbf{J}^T$ is $n \times 1$. The sufficient condition for rectifying the negative tension of particular solution to positive tension is the existence of a null space vector $\mathbf{n}$ of $\mathbf{J}^T$ with all positive entries, e.g., refer to (Roberts et al., 1998). In the presence of external wrench, even if there is no null space vector of $\mathbf{J}^T$ with consistent sign, positive wire tension is feasible if there exist a null space vector $\mathbf{n}_{aug}$ of the augmented Jacobian matrix $\mathbf{J}_{aug}^T$ with non-negative values for the first n entries (corresponding to wires) and positive values for the last t entries (corresponding to non-zero components of external wrench). Detailed discussion on the conditions for positive tension in wires are presented in (Notash, 2012a).

2.2 Methodology for Adjusting Negative Wire Tension

When $n = m+1$ and the pose is in the wrench closure workspace, if the minimum norm solution results in negative tension for wire i, i.e., $\tau_{pi} < 0$, wire i could be considered as "failed" and its tension should be set to a non-negative value τ_{ci}. If wire i is left as slack (Figure 2) $\tau_{ci} = 0$. To increase the wrench capability and stiffness of manipulators, the tension of wire i could be adjusted to a positive value, $\tau_{ci} > 0$. Rewriting equation (2) for wire i

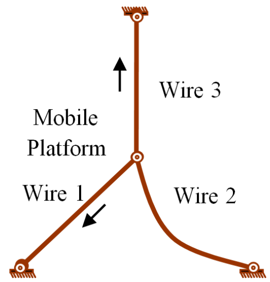

Fig. 2 Three-wire-actuated parallel manipulator with slack wire 2.

$$\tau_{pi} + \tau_{hi} = \tau_{pi} + n_i\lambda = \tau_{ci} \geq 0 \tag{3}$$

or $n_i\lambda = \tau_{ci} + |\tau_{pi}|$, where n_i corresponds to entry i of the null space vector $\mathbf{n}$ and $|\tau_{pi}|$ is the magnitude (absolute value) of τ_{pi}. Then, the platform wrench becomes

$$\mathbf{F}_f = \left[\mathbf{J}_1^T\ \mathbf{J}_2^T\ \cdots\ \mathbf{J}_i^T\ \cdots\ \mathbf{J}_{n-1}^T\ \mathbf{J}_n^T\right]\boldsymbol{\tau}_f \ = \sum_{j=1}^{n}\mathbf{J}_j^T\tau_{pj} - \mathbf{J}_i^T(\tau_{pi} - \tau_{ci}) \tag{4}$$

where $\boldsymbol{\tau}_f = [\tau_{p1}\ \tau_{p2}\ \ldots\ \tau_{ci}\ \ldots\ \tau_{p\,n-1}\ \tau_{p\,n}]^T$ and the change in tension of wire i after adjusting its negative value is $|\tau_{pi} - \tau_{ci}|$. To provide the platform wrench $\mathbf{F}$, the remaining wires must balance the wrench corresponding to the adjusted negative wire tension. With the "correctional" force provided by the remaining wires $\boldsymbol{\tau}_{corr} = [\tau_{corr\,1}\ \tau_{corr\,2}\ \ldots\ 0\ \ldots\ \tau_{corr\,n-1}\ \tau_{corr\,n}]^T$, the recovered wrench will be

$$\mathbf{F}_r = \mathbf{J}^T\boldsymbol{\tau}_f + \mathbf{J}_f^T\boldsymbol{\tau}_{corr} \tag{5}$$

where column i of $\mathbf{J}_f^T = \left[\mathbf{J}_1^T\ \mathbf{J}_2^T\ \cdots\ \mathbf{0}\ \cdots\ \mathbf{J}_{n-1}^T\ \mathbf{J}_n^T\right]$ and entry i of $\boldsymbol{\tau}_{corr}$ are replaced by zeros. Then, the change in the platform wrench will be $\mathbf{F} - \mathbf{F}_r = \mathbf{J}^T(\boldsymbol{\tau}_p - \boldsymbol{\tau}_f) - \mathbf{J}_f^T\boldsymbol{\tau}_{corr}$. When the minimum 2-norm solution results in negative tension for g wires, after adjusting the negative tensions to positive values, the platform wrench that should be balanced by the remaining wires is $\sum_g \mathbf{J}_i^T(\tau_{pi} - \tau_{ci}) = \mathbf{J}^T(\boldsymbol{\tau}_p - \boldsymbol{\tau}_f)$, where the summation is taken over the wires with negative tension.

To fully compensate for the adjusted negative tensions, i.e., for $\mathbf{F} - \mathbf{F}_r = \mathbf{0}$, the correctional force provided by the remaining wires should be (Notash, 2012a)

$$\boldsymbol{\tau}_{corr} = \mathbf{J}_f^{\#T}\sum \mathbf{J}_i^T(\tau_{pi} - \tau_{ci}) = \mathbf{J}_f^{\#T}\mathbf{J}^T(\boldsymbol{\tau}_p - \boldsymbol{\tau}_f) \tag{6}$$

where g columns of $\mathbf{J}^T$, corresponding to the wires with negative tension, are replaced by zeros resulting in $\mathbf{J}_f^T$. Then, the overall wire force will be

$$\boldsymbol{\tau}_{tot} = \boldsymbol{\tau}_f + \boldsymbol{\tau}_{corr} = \mathbf{J}_f^{\#T}\mathbf{J}^T\boldsymbol{\tau}_p + (\mathbf{I} - \mathbf{J}_f^{\#T}\mathbf{J}^T)\boldsymbol{\tau}_f \tag{7}$$

These $\boldsymbol{\tau}_{corr}$ and $\boldsymbol{\tau}_{tot}$ are minimum 2-norm solutions for the chosen τ_{ci} (Notash, 2012b). If $\mathbf{J}_f^T$ has full row-rank, i.e., $\mathbf{F}$ belongs to the range space of $\mathbf{J}_f^T$, $\mathbf{F} \in$

$\Re(\mathbf{J}_f^T)$, $\mathbf{F}_{\Re^\perp} = \left(\mathbf{I} - \mathbf{J}_f^T \mathbf{J}_f^{\#T}\right)\mathbf{F} = \mathbf{0}$, the right-generalized inverse (GI) of $\mathbf{J}_f^T$ is $\mathbf{J}_f^{\#T} = \mathbf{J}_f\left(\mathbf{J}_f^T\mathbf{J}_f\right)^{-1}$ as the vector of wire forces is physically consistent. Otherwise, the weighted left-GI of $\mathbf{J}_f^T$ is used to obtain the platform wrench that best approximates the required wrench in the least-square sense.

When the minimum norm solution results in negative tension for g wires and the pose is in the wrench closure workspace $\tau_{pi} + \tau_{hi} = \tau_{ci} \geq \tau_{\min} \geq 0$ for each of g wires, where $\tau_{\min}$ is the minimum allowable tension. In the following subsections, formulations of τ_{ci} are presented for $n = m+1$. The implementation of this methodology for adjusting negative wire tensions when the null space basis of $\mathbf{J}^T$ is spanned by two or more vectors, i.e., when $n > m+1$, is presented in (Notash, 2013).

2.3 Minimum 2-Norm with Negative Wire Tension

Negative tension for one wire. When the pose is in the wrench closure workspace and the minimum norm solution results in negative tension for wire i, $\tau_{pi} < 0$, considering the non-negative null space vector $\mathbf{n}$, $\tau_{pi} + n_i\lambda = \tau_{ci} \geq \tau_{\min}$, there is no condition on τ_{ci} provided the adjusted wire tensions do not exceed the maximum value. The minimum 2-norm solutions for the correctional and overall wire tension vectors are calculated using equations (6) and (7) for the chosen $\tau_{ci} \geq \tau_{\min}$ value.

For the poses that $\mathbf{n}$ has both positive and negative entries, when $\tau_{pi} < 0$ all wire tensions could be adjusted to non-negative values if

$$\frac{\tau_{\min} + |\tau_{pi}|}{n_i} \leq \frac{\tau_{\min} - \tau_{pj}}{n_j} \quad \text{for} \quad n_i > 0 \quad \text{and} \quad n_j < 0 \tag{8}$$

with $\tau_{pj} > 0$ for $j \neq i$. That is, when the condition on $\mathbf{n}_{aug}$ is met and $\tau_{pi} < 0$ the wire tensions could be set to positive values using $\tau_{ci} \geq \tau_{\min}$.

Negative tension for two or more wires. When the pose is in the wrench closure workspace and the minimum norm solution results in negative tension for wires i and j the non-negative values for these two wires, i.e., $\tau_{ci} \geq \tau_{\min}$ and $\tau_{cj} \geq \tau_{\min}$, cannot be selected arbitrarily. Using a non-negative null space vector $\mathbf{n}$

$$\begin{aligned} \lambda_i &\geq \tfrac{\tau_{\min} - \tau_{pi}}{n_i} \qquad \text{for} \quad n_i > 0 \\ \lambda_j &\geq \tfrac{\tau_{\min} - \tau_{pj}}{n_j} \qquad \text{for} \quad n_j > 0 \end{aligned} \tag{9}$$

The largest of λ_i and λ_j corresponds to the dominating wire and its τ_c is set to $\tau_{\min}$. The adjusted tension of non-dominating wire, e.g., wire j, is calculated using $\tau_{cj} = \tau_{pj} + n_j\lambda_{dw} > \tau_{\min}$. Equation (9) is also valid when $\mathbf{n}$ has both positive and negative entries with $n_i > 0$ and $n_j > 0$. It should be noted that when n_i and n_j have opposite signs, e.g., $n_i > 0$ and $n_j < 0$, the pose is not in the wrench closure workspace as $\lambda_i > 0$ and $\lambda_j < 0$. When $n_i = n_j$, the dominating wire l_{dw} is the wire corresponding to max $(|\tau_{pi}|, |\tau_{pj}|)$. Then, the minimum 2-norm solutions for the correctional and overall wire tension vectors are calculated using equations (6) and (7) for the chosen τ_c values.

The dominating wire can also be identified using $Q = (\tau_{pi}n_j - \tau_{pj}n_i)/(n_j - n_i)$. For example, when $n_i \neq n_j$, provided $\tau_{pj} + n_j\lambda_{dw} > \tau_{\min}$

$$Q = \frac{\tau_{pi}n_j - \tau_{pj}n_i}{n_j - n_i} \leq \tau_{\min} \quad \Rightarrow \quad l_{dw} = i \quad , \quad \tau_{ci} = \tau_{\min} \tag{10}$$

The procedure could be generalized for the case that the minimum 2-norm solution results in negative tension for g wires to identify the dominating wire as

$$\lambda_{dw} = \max\,(\lambda_i, \lambda_j, \cdots) = \max\left(\frac{\tau_{\min} + |\tau_{pi}|}{n_i}, \frac{\tau_{\min} + |\tau_{pj}|}{n_j}, \cdots\right) \tag{11}$$

Minimum norm positive tension within upper limit. To ensure the adjusted non-negative wire tensions do not exceed the maximum allowable tension, i.e., $\tau_{pl} + n_l\lambda_{dw} \leq \tau_{\max}$, for $l = 1, \cdots, n$, the following conditions should be satisfied.

When the entries of the null space vector $\mathbf{n}$ have consistent signs (non-negative), while adjusting the negative tension of wires, e.g., wire i with $\tau_{pi} + n_i\lambda \geq \tau_{\min}$, the tension of wires with positive particular solution will be increased (will remain unchanged if the corresponding entry of $\mathbf{n}$ is zero). When wire k has the smallest $\lambda_k = (\tau_{\max} - \tau_{pk})/n_k$ among all wires with positive particular solution the adjusted tension of wires will not exceed the limit as long as

$$0 < \frac{\tau_{\min} + |\tau_{pi}|}{n_i} \leq \frac{\tau_{\max} - \tau_{pk}}{n_k} \quad \text{for} \quad n_i > 0 \quad \text{and} \quad n_k > 0 \tag{12}$$

When more than one wire has negative tension, in equation (12) wire i corresponds to the dominating wire of equation (11). Hence, the sufficient condition for exceeding the upper limit is $\tau_{pk} \geq \tau_{\max}$, while the necessary condition is $\tau_{pk} > \tau_{\max} - n_k\lambda_{dw}$.

When n_i and n_k have opposite signs, e.g., $n_i > 0$ and $n_k < 0$, for $\tau_{pk} + n_k\lambda_{dw} \leq \tau_{\max}$, the limit on maximum tension is satisfied as long as $\tau_{pk} \leq \tau_{\max} + |n_k|\,\lambda_{dw}$.

3 Case Study

In the planar wire-actuated parallel manipulators, the mobile platform is connected to the base by n wires, each wire with a length of l_i and orientation of α_i (Figure 3). The attachment points of wire i to the base and platform are denoted as points A_i and B_i, respectively. The angular positions of points B_i on the platform are denoted by θ_i. For a 2 DOF translational manipulator with three wires, the coordinates of A_i, $i = 1, \ldots, 3$, are (–2, –1.5), (2, –1.5) and (0, 1.5), respectively, and points B_i coalesce. At the platform pose of $\mathbf{p} = [0.5\ \ -0.5]^T$ meters, which is in the wrench

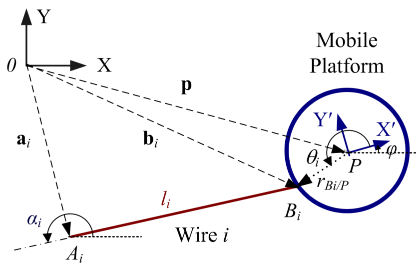

Fig. 3 Parameters of planar wire-actuated parallel manipulators.

closure workspace, the wire forces $\boldsymbol{\tau}$ are related to the platform wrench $\mathbf{F}$ using $\mathbf{F} = \mathbf{J}^T\boldsymbol{\tau}$, where

$$\mathbf{J}^T = \begin{bmatrix} \cos\alpha_1 & \cos\alpha_2 & \cos\alpha_3 \\ \sin\alpha_1 & \sin\alpha_2 & \sin\alpha_3 \end{bmatrix} = \begin{bmatrix} -0.929 & 0.832 & -0.243 \\ -0.371 & -0.555 & 0.970 \end{bmatrix} \tag{13}$$

For $\mathbf{F} = [-3.309\ 14.737]^T$ Newtons, the minimum norm vector of wire forces is $\boldsymbol{\tau}_p = \mathbf{J}^{\#T}\mathbf{F} = [-4.236\ \ -5.699\ \ 10.310]^T$ with a magnitude of $\left\|\boldsymbol{\tau}_p\right\|_2 = 12.519$ and negative tension for wires 1 and 2. Then, the 2×5 $\mathbf{J}_{aug}^T$ is

$$\mathbf{J}_{aug}^T = \left[\mathbf{J}_1^T\ \mathbf{J}_2^T\ \mathbf{J}_3^T\ \mathbf{W}_1^T\mathbf{W}_2^T\right] = \begin{bmatrix} -0.929 & 0.832 & -0.243 & 3.309 & 0 \\ -0.371 & -0.555 & 0.970 & 0 & -14.737 \end{bmatrix} \tag{14}$$

A non-negative null space vector of $\mathbf{J}_{aug}^T$ is $\mathbf{n}_{aug} = [0.413\ \ 0.576\ \ 9.981\ \ 0.703\ \ 0.625]^T$. The non-negative null space vector of $\mathbf{J}^T$ is $\mathbf{n} = [0.463\ \ 0.682\ \ 0.567]^T$, with non-zero entries corresponding to wires 1 and 2. As $\frac{\tau_{p1}n_2 - \tau_{p2}n_1}{n_2 - n_1} = -1.143 < \tau_{\min} = 1$, wire 1 is the dominating wire and its tension is adjusted to $\tau_{c1} = 1$ N and the tension of wire 2 is calculated as $\tau_{c2} = 2.014$ N. These adjusted tensions correspond to λ =max($\lambda_1 = 11.315$, $\lambda_2 = 9.829$). Then

$$\boldsymbol{\tau}_{corr} = \mathbf{J}_f^{\#T} \sum \mathbf{J}_i^T(\tau_{pi} - \tau_{ci}) = [0\ \ 0\ \ 6.414]^T \tag{15}$$

$$\boldsymbol{\tau}_{tot} = \boldsymbol{\tau}_f + \boldsymbol{\tau}_{corr} = [1.000\ \ 2.014\ \ 16.724]^T \tag{16}$$

which produces the original wrench, and $\|\boldsymbol{\tau}_{corr}\|_2 = 6.414$, $\|\boldsymbol{\tau}_{tot}\|_2 = 16.875$.

At the platform pose of $\mathbf{p} = [2\quad 0]^T$ meters, wire 2 is in Y direction, and for $\mathbf{F} = [-17.363\ \ 2.489]^T$ Newtons, the minimum norm vector of wire forces is $\boldsymbol{\tau}_p = \mathbf{J}^{\#T}\mathbf{F} = [10.490\ \ -0.516\ \ 9.427]^T$ with a magnitude of $\left\|\boldsymbol{\tau}_p\right\|_2 = 14.113$ and negative tension for wire 2, using

$$\mathbf{J}^T = \begin{bmatrix} -0.936 & 0 & -0.800 \\ -0.351 & -1.000 & 0.600 \end{bmatrix} \tag{17}$$

The null space vector of $\mathbf{J}^T$ is $\mathbf{n} = [-0.536\ \ 0.565\ \ 0.628]^T$, with a negative entry corresponding to wire 1. Therefore, in the absence of external force this pose is not within the wrench closure workspace. A non-negative null space vector of $\mathbf{J}_{aug}^T$ is $\mathbf{n}_{aug} = [0.680\ \ 0.074\ \ 0.725\ \ 0.070\ \ 0.049]^T$, hence wire tensions could be adjusted to positive. This is also evident using condition (8)

$$\frac{\tau_{\min} + |\tau_{p2}|}{n_2} = 2.684 \leq \frac{\tau_{\min} - \tau_{p1}}{n_1} = 17.702 \tag{18}$$

The tension of wire 2 is set to $\tau_{c2} = 1$ N, which corresponds to $\lambda = 2.684$. Then

$$\boldsymbol{\tau}_{corr} = \mathbf{J}_f^{\#T} \mathbf{J}_2^T (\tau_{p2} - \tau_{c2}) = [-1.439 \quad 0 \quad 1.684]^T \tag{19}$$

$$\boldsymbol{\tau}_{tot} = \boldsymbol{\tau}_f + \boldsymbol{\tau}_{corr} = [9.051 \quad 1.000 \quad 11.111]^T \tag{20}$$

which produces the original wrench, and $\|\boldsymbol{\tau}_{corr}\|_2 = 2.215$, $\|\boldsymbol{\tau}_{tot}\|_2 = 14.366$.

4 Conclusion

For wire-actuated parallel manipulators, the minimum 2-norm solution for the vector of wire tensions could result in negative tension for one or more wires. The negative tensions could be adjusted to positive values using the null space vector of the transposed Jacobian matrix and adding the homogeneous solution to the particular solution. In this paper, a methodology for adjusting the negative tension of the minimum 2-norm solution using the generalized inverse of the transposed Jacobian matrix was presented. The method results in minimum 2-norm positive (non-negative) solution for wire tension vector while satisfying the upper limit on tension. The implementation of the methodology was illustrated on a 2 DOF translational manipulator.

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